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Down-conversion of a single photon with unit efficiency

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The down-conversion dynamics of a single photon in a single three-level system are investigated theoretically. The wave functions of the output photons are obtained analytically as functions of the input photon profile, and the down-conversion probability is evaluated from these output wave functions. It is demonstrated that down-conversion of a single photon with unit efficiency is possible and the necessary conditions are clarified.

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I. INTRODUCTION

Parametric down-conversion is a representative second-order nonlinear optical phenomenon in which a pump beam is converted into signal and idler beams having lower frequencies. As a phenomenon of classical nonlinear optics, the dynamics of down-conversion have been successfully described by coupled-mode theory, which is based on the linear and nonlinear optical susceptibilities and the classical Maxwell equations [1–3]. However, the interest in down-conversion dynamics is continuing to grow in the field of quantum optics, since parametric down-conversion is one of the most popular physical process for generating nonclassical states of light, such as squeezed states and entangled twin photons [4–8]. In particular, extensive efforts are being made to enhance the efficiency of twin-photon generation [9–13], since twin photons are essential in fundamental tests of quantum mechanics [14,15] and also in quantum technologies [16–20].

A standard method for generating down-converted twin photons is to illuminate a $\chi^{(2)}$ material with classical light pulses. Since photons in classical light pulses are independent, the key to controlling the down-conversion efficiency lies in gaining a complete understanding of how a single parent photon is converted into two daughter photons. For this purpose, quantum-optical analyses of the down-conversion process are required, and such analyses have been performed by using phenomenological interaction Hamiltonians under the single-mode approximation [21,22] and by using multimode perturbative treatments [23–25]. In these analyses, optical materials are treated implicitly through the coupling constants between the photonic modes. However, in order to handle the quantum coherence interchanged between photons and materials correctly and therefore to evaluate the generated twin photons quantitatively, the full-quantum treatment for both photons and materials becomes essential, particularly when the input photons are close to the resonance of the materials.

The objective of this study is to investigate the down-conversion dynamics of a input photon based on the full-quantum multimode formalism. Using a three-level system as the simplest $\chi^{(2)}$ system, the wave function of the down-converted photons is obtained in an analytic form, and the down-conversion probability is calculated from it. As a result, it is shown that there exists a condition under which the input photon is down-converted with unit efficiency. The insights into fundamental single-photon dynamics reported here will serve as guidelines for developing effective photon manipulation techniques.

II. SYSTEM

A. Hamiltonian

As illustrated in Fig. 1, the system considered in this study consists of a one-dimensional photon field and a three-level system (referred to hereafter as an "atom") located at $r=0$ [26,27]. The three quantum levels of the atom are denoted by $|g\rangle$, $|m\rangle$, and $|e\rangle$, as shown in Fig. 1. Putting $\hbar=c=1$, the Hamiltonian for the overall system is given, under the rotating wave approximation, by

$$\mathcal{H} = \Omega_e\sigma_{ee} + \Omega_m\sigma_{mm} + \int dk k a_k^\dagger a_k + [i(\sqrt{\Gamma_1}\sigma_{eg} + \sqrt{\Gamma_2}\sigma_{em} + \sqrt{\Gamma_3}\sigma_{mg})\tilde{a}_{r=0} + \text{H.c.}], \quad (1)$$

where the atomic transition operator is defined by $\sigma_{ij}=|i\rangle\langle j|$

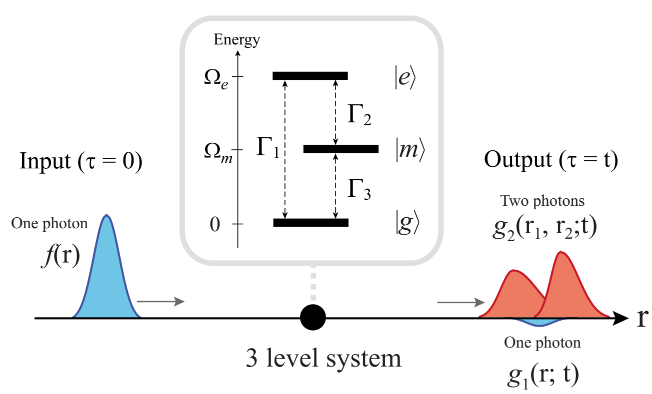

FIG. 1. (Color online) Schematic illustration of the situation considered in this study. A parent photon resonant to the $|g\rangle\rightarrow|e\rangle$ transition is input into a three-level atom and is down-converted into two daughter photons with some probability. Under certain conditions, the parent photon can be down-converted with unit efficiency.

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$(i,j=e,g,m)$, a_k is the photon annihilation operator with wave number k, and $\tilde{a}_{r=0}$ is the photon annihilation operator at $r=0$. a_k and $\tilde{a}_r$ are related by the Fourier transformation as

$$\tilde{a}_r = (2\pi)^{-1/2} \int dk e^{ikr} a_k. \quad (2)$$

The parameters are defined as follows: Ω_e and Ω_m are the energies of $|e\rangle$ and $|m\rangle$ measured from $|g\rangle$, respectively, and Γ_1, Γ_2, and Γ_3 are the decay rates for the $|e\rangle \to |g\rangle$, $|e\rangle \to |m\rangle$ and $|m\rangle \to |g\rangle$ transitions, respectively.

B. Input and output photons

Initially $(\tau=0)$, the atom is in the ground state and a single photon resonant to the $|g\rangle \to |e\rangle$ transition is input from the left-hand side (i.e., the $r<0$ region). The input state vector is given by

$$|\psi_{\rm in}\rangle = \int dr f(r) \tilde{a}_r^\dagger |0\rangle, \quad (3)$$

where $f(r)$ is the wave function of the single input photon, which is normalized as $\int dr |f(r)|^2 = 1$ and vanishes in the $r>0$ region, and $|0\rangle = |g\rangle \otimes |v\rangle$, where $|v\rangle$ is the photonic vacuum state. After the interaction with the atom $(\tau=t)$, a two-photon component may be generated in addition to a one-photon component, due to the cascade decay channel of $|e\rangle \to |m\rangle \to |g\rangle$. The output state vector may therefore be written as

$$|\psi_{\rm out}\rangle = \int dr g_1(r;t) \tilde{a}_r^\dagger |0\rangle + \int\int dr_1 dr_2 \frac{g_2(r_1,r_2;t)}{\sqrt{2}} \tilde{a}_{r_1}^\dagger \tilde{a}_{r_2}^\dagger |0\rangle, \quad (4)$$

where g_1 and g_2 are the one- and two-photon output wave functions, respectively. The norms of these wave functions, $P_1 = \int dr |g_1(r;t)|^2$ and $P_2 = \int\int dr_1 dr_2 |g_2(r_1,r_2;t)|^2$, respectively represent the probabilities for one- and two-photon output and satisfy $P_1 + P_2 = 1$. In the following part of this study, we calculate the output wave function g_2 and from it determine the down-conversion probability P_2.

III. RELATION BETWEEN INPUT AND OUTPUT WAVE FUNCTIONS

The input and output state vectors are related through the Schrödinger equation

$$|\psi_{\rm out}\rangle = e^{-i\mathcal{H}t} |\psi_{\rm in}\rangle. \quad (5)$$

As shown in the Appendix, we can analytically solve this equation by considering a coherent state as the input state [28,29]. The relation between the input wave function $[f(r)]$ and the one- and two-photon output wave functions $[g_1(r;t)$ and $g_2(r_1,r_2;t)]$ is summarized by the following equations:

$$g_1(r;t) = f(r-t) - \sqrt{\Gamma_1} \langle \sigma_{ge}(t-r) \rangle^L, \quad (6)$$

$$g_2(r_1,r_2;t) = \sqrt{\frac{\Gamma_2 \Gamma_3}{2}} e^{(i\Omega_m + \Gamma_3/2)(r_s - r_l)} \langle \sigma_{ge}(t-r_l) \rangle^L, \quad (7)$$

$$\langle \sigma_{ge}(t) \rangle^L = \sqrt{\Gamma_1} \int_0^\infty d\xi f(-t+\xi) e^{-[i\Omega_e + (\Gamma_1+\Gamma_2)/2]\xi}, \quad (8)$$

where r_l and r_s in Eq. (7) are the space coordinates for the preceding and succeeding photons, defined by $r_l = \max(r_1,r_2)$ and $r_s = \min(r_1,r_2)$, respectively, and $\langle \sigma_{ge}(t) \rangle^L$ is the atomic response induced by the input photon.

IV. NUMERICAL RESULTS

In this section, we visualize the analytic results of Eqs. (6)–(8) by numerically calculating the down-conversion probability and the pulse profiles of output photons. To be more specific, we hereafter assume that the input photon is in resonance with the $|g\rangle \to |e\rangle$ transition and has a Gaussian mode function with pulse length d. Denoting the initial position of the pulse by a $(a<0$ and $|a| \gg d)$, the input mode function is given by

$$f(r) = \left(\frac{2}{\pi d^2}\right)^{1/4} \exp\left[-\left(\frac{r-a}{d}\right)^2 + i\Omega_e(r-a)\right]. \quad (9)$$

From Eq. (8), the atomic response is given by

$$\langle \sigma_{ge}(t) \rangle^L = \left(\frac{\pi \Gamma_1^2 d^2}{8}\right)^{1/4} \exp\left[\frac{(\Gamma_1 d + \Gamma_2 d)^2}{16} - \left(i\Omega_e + \frac{\Gamma_1+\Gamma_2}{2}\right) \times (t+a)\right] \operatorname{erfc}\left[\frac{(\Gamma_1+\Gamma_2)d}{4} - \frac{t+a}{d}\right], \quad (10)$$

where the complementary error function is defined by $\operatorname{erfc}(x) = 2\pi^{-1/2}\int_x^\infty d\xi \exp(-\xi^2)$. The one-photon wave function g_1 is given by Eq. (6). This photon is located at $r \simeq a+t$, and its central frequency is Ω_e. The two-photon wave function g_2 is given by Eq. (7). These photons are also located at $r \simeq a+t$, and the central frequencies of the preceding and succeeding photons are $\Omega_e - \Omega_m$ and Ω_m, respectively, as expected by the atomic energy diagram shown in Fig. 1.

A. Down-conversion probability

First, we investigate the down-conversion probability P_2, which is given, as the norm of the two-photon wave function, by

$$P_2 = \int dr_1 dr_2 |g_2(r_1,r_2;t)|^2. \quad (11)$$

From Eq. (7), we obtain $P_2 = \Gamma_2 \int d\tau |\langle \sigma_{ge}(\tau) \rangle^L|^2$. Therefore, P_2 is determined solely by the atomic response $\langle \sigma_{ge}(\tau) \rangle^L$ of the atom, and is independent of Ω_m (the energy of $|m\rangle$) and Γ_3 (the decay rate of the $|m\rangle \to |g\rangle$ transition). In Fig. 2, P_2 is plotted as a function of the coherence length d of the input photon for several values of Γ_2/Γ_1. As Fig. 2 shows, P_2 is a monotonically increasing function of d, indicating that a longer pulse is more advantageous for down-conversion. In the large d region satisfying $d \gg \Gamma^{-1}$, the down-conversion probability P_2 can be approximated by

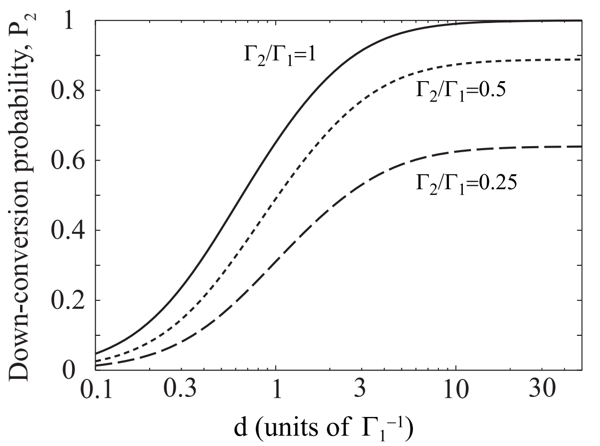

FIG. 2. Down-conversion probability P_2 as a function of the coherence length d of the input photon. Γ_2/Γ_1 is set at 1 (solid line), 0.5 (dotted line), and 0.25 (dashed line).

$$P_2 \simeq \frac{4\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^2}. \tag{12}$$

From the inequality of the arithmetic and geometric means, P_2 takes a maximum value of unity when $\Gamma_1=\Gamma_2$. Thus, down-conversion with unit efficiency ($P_2 \simeq 1$) is possible when the following conditions are satisfied: (i) the decay rates for the $|e\rangle \to |g\rangle$ and $|e\rangle \to |m\rangle$ transitions are equal ($\Gamma_1=\Gamma_2$) and (ii) the input pulse is sufficiently long ($d \gg \Gamma^{-1}$).

Let us briefly discuss here the key mechanism of down-conversion with unit efficiency. As observed in Eq. (6), the one-photon output is composed of the transmitted photon and the atomic emission associated with the $|e\rangle \to |g\rangle$ transition. When the input pulse is sufficiently long, it can be approximated by a stationary wave as $f(r) \sim E e^{i\Omega_e r}$. In this case, the transmitted photon is given by $f(r-t) \sim E e^{i\Omega_e(r-t)}$, whereas the atomic emission is given, from Eq. (8), by $-\sqrt{\Gamma_1}\langle \sigma_{ge}(t-r)\rangle^L \sim -2\Gamma_1/(\Gamma_1+\Gamma_2)E e^{i\Omega_e(r-t)}$. Thus, these two components interfere destructively, and the down-conversion probability reaches unity when $\Gamma_1=\Gamma_2$.

B. Profiles of output photons

Next, we observe the shape of the output photons when the down-conversion probability is almost unity. Here, the output pulse shapes are characterized by the intensity distribution $I^{(n)}_{\rm out}(r;t) = \langle \psi^{(n)}_{\rm out}|\tilde{a}^\dagger_r \tilde{a}_r|\psi^{(n)}_{\rm out}\rangle / n$, where n denotes the photon number and $|\psi^{(n)}_{\rm out}\rangle$ is the n-photon component of the output state vector. This intensity distribution function is normalized as $P_n = \int dr I^{(n)}_{\rm out}(r;t)$, where P_n is the n-photon probability in the output. From Eq. (4), we obtain $I^{(1)}_{\rm out}(r;t) = |g_1(r;t)|^2$ and $I^{(2)}_{\rm out}(r;t) = \int dr' |g_2(r,r';t)|^2$. These functions are plotted in Fig. 3, together with the input pulse profile given by $I_{\rm in}(r) = |f(r)|^2$ for reference. It can be observed that the down-converted photons are slightly delayed relative to the input pulse. The delay time is of the order of Γ^{-1}, which is required for absorption and re-emission by the atom.

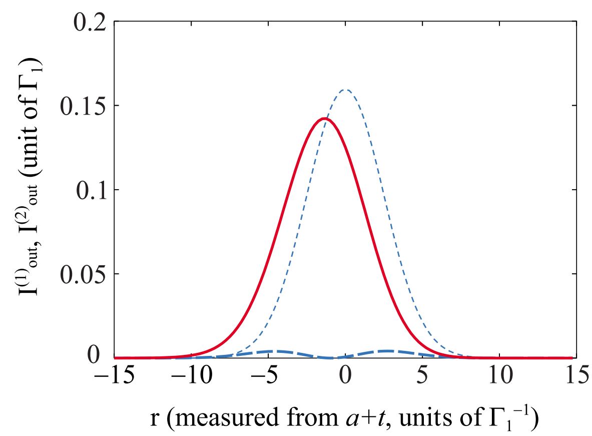

FIG. 3. (Color online) Intensity profiles of the one-photon output (dashed line) and the two-photon output (solid line). The input photon profile (thin dotted line) is also plotted for reference. The parameters are chosen as $\Gamma_1=\Gamma_2=\Gamma_3$ and $d=5/\Gamma_1$. Under this condition, the down-conversion probability P_2 reaches 96.4%.

C. Remarks

A final remark is in order. The three-level atom is assumed to be motionless in this study. We briefly observe here the effect of atomic motion. The atomic motion changes the coupling constants Γ_1 and Γ_2 through the photonic mode function, but the ratio Γ_1/Γ_2 is kept unchanged. On the other hand, as observed in Fig. 2, the down-conversion probability P_2 depends only on Γ_1/Γ_2 in the large d region. Thus, the current results are expected to be robust against the atomic motion, as long as the coherence length of the input photon is large.

V. SUMMARY

In summary, the down-conversion dynamics of a single photon in a three-level atom is investigated theoretically. The one- and two-photon output wave functions are obtained analytically as functions of the input photon profile, and the down-conversion probability is evaluated from them. It is demonstrated that a single photon can be down-converted with almost unit efficiency, provided that the decay rates for the $|e\rangle \to |g\rangle$ and $|e\rangle \to |m\rangle$ transitions are identical (see Fig. 1), and that the input photon has a sufficiently long coherence length. Atomic systems in combination with the electromagnetically induced transparency effects would be promising to realize such situations.

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APPENDIX: SOLUTION OF SCHRÖDINGER EQUATION

The output state vector can be obtained by solving the Schrödinger equation (5). However, instead of solving this equation directly, it is more convenient to use a coherent state $|\phi_{\rm in}\rangle$ as the input state [28,29]. The state vector of a coherent state is given by

$$|\phi_{\rm in}\rangle = \mathcal{N}\exp\left[\mu\int dr f(r)\tilde{a}_r^\dagger\right]|0\rangle, \tag{A1}$$

where the normalization constant is given by $\mathcal{N}=e^{-|\mu|^2/2}$. By expanding Eq. (A1) in powers of μ as $|\phi_{\rm in}\rangle=\mathcal{N}[|0\rangle+\mu|\psi_{\rm in}\rangle+\mathcal{O}(\mu^2)]$, it is observed that the linear component of $|\phi_{\rm in}\rangle$ is the single-photon state $|\psi_{\rm in}\rangle$. (In this study, quantities proportional to μ are referred to as "linear" quantities.) The output state vector for $|\phi_{\rm in}\rangle$ is given by $|\phi_{\rm out}\rangle=e^{-i\mathcal{H}t}|\phi_{\rm in}\rangle=\mathcal{N}[|0\rangle+\mu|\psi_{\rm out}\rangle+\mathcal{O}(\mu^2)]$. The one- and two-point correlation functions for $|\phi_{\rm out}\rangle$ are defined by $\mathcal{G}_1(r;t)=\langle\phi_{\rm out}|\tilde{a}_r|\phi_{\rm out}\rangle$ and $\mathcal{G}_2(r_1,r_2;t)=\langle\phi_{\rm out}|\tilde{a}_{r_1}\tilde{a}_{r_2}|\phi_{\rm out}\rangle$. In particular, their linear components (denoted by $\mathcal{G}_1^L$ and $\mathcal{G}_2^L$) are relevant for our purpose, because they are related to g_1 and g_2 by the following relations:

$$\mathcal{G}_1^L(r;t) = \langle 0|\tilde{a}_r|\psi_{\rm out}\rangle = g_1(r;t), \tag{A2}$$

$$\mathcal{G}_2^L(r_1,r_2;t) = \langle 0|\tilde{a}_{r_1}\tilde{a}_{r_2}|\psi_{\rm out}\rangle = \sqrt{2}g_2(r_1,r_2;t). \tag{A3}$$

Thus, the output wave functions g_1 and g_2 are calculated by the following two steps: First, assuming a coherent-state input of Eq. (A1), we calculate $\mathcal{G}_1^L$ and $\mathcal{G}_2^L$ (the linear components of the correlation functions) in the output field. Then, we use Eqs. (A2) and (A3) to obtain g_1 and g_2.

In order to calculate $\mathcal{G}_1^L$ and $\mathcal{G}_2^L$, we work in the Heisenberg picture. The initial state vector is given by Eq. (A1). The Heisenberg equations for σ_{gm} and σ_{ge} are given by

$$\frac{d}{d\tau}\sigma_{gm} = \left(-i\Omega_m - \frac{\Gamma_3}{2}\right)\sigma_{gm} + \sqrt{\Gamma_1\Gamma_2}\sigma_{ee} + \sqrt{\Gamma_2\Gamma_3}\sigma_{me} - \frac{\sqrt{\Gamma_1\Gamma_3}}{2}\sigma_{ge} - \sqrt{\Gamma_1}\sigma_{em}\tilde{a}_{-\tau}(0) - \sqrt{\Gamma_2}\tilde{a}_{-\tau}^\dagger(0)\sigma_{ge} + \sqrt{\Gamma_3}(\sigma_{gg}-\sigma_{mm})\tilde{a}_{-\tau}(0), \tag{A4}$$

$$\frac{d}{d\tau}\sigma_{ge} = \left(-i\Omega_e - \frac{\Gamma_1+\Gamma_2}{2}\right)\sigma_{ge} - \frac{\sqrt{\Gamma_1\Gamma_3}}{2}\sigma_{gm} + \sqrt{\Gamma_1}(\sigma_{gg}-\sigma_{ee})\tilde{a}_{-\tau}(0) + \sqrt{\Gamma_2}\sigma_{gm}\tilde{a}_{-\tau}(0) - \sqrt{\Gamma_3}\sigma_{me}\tilde{a}_{-\tau}(0), \tag{A5}$$

where $\tilde{a}_{-\tau}(0)$ is the initial field operator with the space coordinate $r=-\tau(<0)$. The output field operator is given by

$$\tilde{a}_r(t) = \tilde{a}_{r-t}(0) - \sqrt{\Gamma_1}\sigma_{ge}(t-r) - \sqrt{\Gamma_2}\sigma_{me}(t-r) - \sqrt{\Gamma_3}\sigma_{gm}(t-r), \tag{A6}$$

where $0<r<t$. Equations (A4)–(A6) are derivable from Eq. (1). The following two properties are frequently used in the subsequent arguments: (i) $|\phi_{\rm in}\rangle$ is an eigenstate of the initial field operator $\tilde{a}_r(0)$, satisfying $\tilde{a}_r(0)|\phi_{\rm in}\rangle=\mu f(r)|\phi_{\rm in}\rangle$. Note that $\mu f(r)$ is a c number, whereas $\tilde{a}_r(0)$ is an operator. (ii) The initial field operator $\tilde{a}_r(0)$ commutes with any atomic operator $\sigma(\tau)$ if $r<-\tau$. The one- and two-point correlation functions are given by $\mathcal{G}_1(r;t)=\langle\tilde{a}_r(t)\rangle$ and $\mathcal{G}_2(r_1,r_2;t)=\langle\tilde{a}_{r_1}(t)\tilde{a}_{r_2}(t)\rangle$, where the initial-state average $\langle\phi_{\rm in}|A|\phi_{\rm in}\rangle$ is denoted by $\langle A\rangle$.

The one-photon output wave function g_1 is obtained as follows. It is confirmed from Eq. (A6) that $\mathcal{G}_1^L$ is composed of four terms as $\mathcal{G}_1^L=f-\sqrt{\Gamma_1}\langle\sigma_{ge}\rangle^L-\sqrt{\Gamma_2}\langle\sigma_{me}\rangle^L-\sqrt{\Gamma_3}\langle\sigma_{gm}\rangle^L$, where $\langle\sigma\rangle^L$ denotes the linear component of $\langle\sigma\rangle$. However, $\langle\sigma_{me}\rangle$ has no linear components. Furthermore, since the input photon is resonant to the $|g\rangle\to|e\rangle$ transition, $\langle\sigma_{gm}\rangle^L$ is highly off-resonant and is negligible. Remembering that the one-photon output wave function g_1 is identical to $\mathcal{G}_1^L$ [see Eq. (A2)], we have

$$g_1(r;t) = f(r-t) - \sqrt{\Gamma_1}\langle\sigma_{ge}(t-r)\rangle^L,$$

which has already been presented as Eq. (6). The equation of motion for $\langle\sigma_{ge}\rangle^L$ is obtained from Eqs. (A1) and (A5). Since only $\langle\sigma_{gg}\rangle$ is nonzero in the zeroth order, the linear-response equation is given by

$$\frac{d}{d\tau}\langle\sigma_{ge}\rangle^L = \left(-i\Omega_e - \frac{\Gamma_1+\Gamma_2}{2}\right)\langle\sigma_{ge}\rangle^L + \sqrt{\Gamma_1}f(-\tau). \tag{A7}$$

The formal solution of this equation is given by Eq. (8).

The two-photon output wave function g_2 is obtained as follows. It is confirmed from Eq. (A6) that $\mathcal{G}_2$ is composed of 16 terms. However, only $\langle\sigma_{gm}\sigma_{me}\rangle$ yields a linear component. Thus, we have

$$\mathcal{G}_2^L(r_1,r_2;t) = \sqrt{\Gamma_2\Gamma_3}\langle\sigma_{gm}(t-r_1)\sigma_{me}(t-r_2)\rangle^L. \tag{A8}$$

Since $\mathcal{G}_2$ is a symmetric function of r_1 and r_2 by definition, we can set $r_1\leqslant r_2$ without loss of generality. The equation of motion for $\langle\sigma_{gm}(\tau)\sigma_{me}(\tau')\rangle$ (satisfying $\tau>\tau'$) is obtained by multiplying Eq. (A4) by $\sigma_{me}(\tau')$ from the right side. Although many terms appear in this equation, only the self-decay term is relevant for the linear dynamics. Namely,

$$\frac{d}{d\tau}\langle\sigma_{gm}(\tau)\sigma_{me}(\tau')\rangle^L = \left(-i\Omega_m - \frac{\Gamma_3}{2}\right)\langle\sigma_{gm}(\tau)\sigma_{me}(\tau')\rangle^L, \tag{A9}$$

with the initial condition of $\langle\sigma_{gm}(\tau')\sigma_{me}(\tau')\rangle^L=\langle\sigma_{ge}(\tau')\rangle^L$. This equation is readily solved to give

$$\langle\sigma_{gm}(\tau)\sigma_{me}(\tau')\rangle^L = \langle\sigma_{ge}(\tau')\rangle^L e^{(i\Omega_m+\Gamma_3/2)(\tau'-\tau)}. \tag{A10}$$

Using Eqs. (A3), (A8), and (A10), the two-photon output wave function is recast into Eq. (7). Thus, Eqs. (6)–(8) connecting the input and output wave functions are derived.

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