Transitional behavior between self-Kerr and cross-Kerr effects by two photons

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(Received 15 January 2007; revised manuscript received 19 March 2007; published 8 June 2007)

The transitional behavior of the two-photon Kerr effect between the self-Kerr and cross-Kerr cases is investigated. To this end, we have developed a semiclassical method for evaluating the two-photon Kerr effect that is applicable to any two-photon input state. It is revealed that the maximum Kerr effect is obtained when the second photon is input with a delay time that corresponds to the absorption time of the first photon by the optical material.

DOI: 10.1103/PhysRevA.75.063807 PACS number(s): 42.65.Hw, 42.50.Dv

I. INTRODUCTION

The interaction of two photons mediated by a nonlinear optical medium has been attracting much attention from the viewpoint of all-optical quantum computation [1–5]. It has been widely regarded that light fields at the single-photon level are too weak to induce significant nonlinear optical effects. However, absorption saturation of a two-level system placed inside an optical cavity has been proven to yield a strong Kerr effect that is sensitive to individual photons [6,7]. Such large Kerr effects can be obtained when the frequencies of the input photons are close to the resonances of the optical system, where the system is highly dispersive and the spatial shapes of photonic pulses are deformed significantly through the interaction. Therefore, in order to utilize this nonlinear effect to construct an all-optical conditional phase gate, control of the spatial profiles of photonic pulses is essential to attain high-fidelity operations.

The Kerr interaction between photons has often been discussed theoretically in terms of phenomenological interaction Hamiltonians, by assuming that the mode functions of photons are unchanged during the interaction (single-mode approximations) [8–14]. In order to construct quantitative theories that can treat changes in photonic pulse profiles and are thus applicable in the near-resonance frequency region, the following two conditions should be satisfied: (i) both the photon field and the optical media must be treated quantum-mechanically (fully quantum-mechanical treatment), and (ii) the multimode nature of the photon field must be rigorously taken into account [15,16]. However, such rigorous analyses usually involve heavy computation and are practically impossible to perform for realistic material systems having many mechanical degrees of freedom. In such systems, the nonlinear dynamics of light have been analyzed using semiclassical theory, in which light fields are treated as classical electromagnetic waves [17,18]. At first glance, semiclassical theory would appear not to be directly applicable to the two-photon problem, since Fock states cannot be described by classical waves. However, it was found that the two-photon Kerr effect can be evaluated in terms of semiclassical results, avoiding the need to resort to fully quantum mechanical calculations [19–21]. While this semiclassical method greatly reduces the computational tasks, it imposes a highly limiting restriction on the two input photons. Denoting the overlap between the mode functions of two input photons by $\mathcal{V}$ [see Eq. (2) for definition], semiclassical evaluation was possible only in the following two limiting cases: (i) the self-Kerr case, in which the two photons are completely identical and therefore $\mathcal{V}=1$ [19], and (ii) the cross-Kerr case, in which the two photons have different polarizations and therefore $\mathcal{V}=0$ [21].

In this study, we investigate the Kerr effect between two photons for the general case of $0 \leqslant |\mathcal{V}| \leqslant 1$. For this purpose, we develop a generalized semiclassical method applicable for any two-photon inputs, namely, any $\mathcal{V}$ (satisfying $0 \leqslant |\mathcal{V}| \leqslant 1$) and any polarizations. Using this method, it is found that the Kerr effect is maximized not in the limiting cases of $|\mathcal{V}|=0$ or 1, but in the transitional situation of $0 < |\mathcal{V}| < 1$. Therefore, in the search for the optimum design of two-photon nonlinear devices, investigation of this transitional region is essential, and the generalized method presented here would be the best approach to solve this problem.

II. QUANTIFICATION OF TWO-PHOTON KERR EFFECT

The situation investigated in this study is illustrated in Fig. 1. Two photons with the same polarization and with different mode functions are input into a Kerr system. The mode functions of the input photons are denoted by $f_a(r)$ and $f_b(r)$, which are normalized as $\int dr|f_a(r)|^2=\int dr|f_b(r)|^2=1$. The input state vector $|AB_{\text{in}}\rangle$ is given by

$$|AB_{\text{in}}\rangle = (1+|\mathcal{V}|^2)^{-1/2} \int dr_1 dr_2 f_a(r_1) f_b(r_2) \tilde{c}^{\dagger}_{r_1} \tilde{c}^{\dagger}_{r_2} |0\rangle, \quad (1)$$

where $\mathcal{V}$ represents the overlap between the mode functions of input photons, as given by

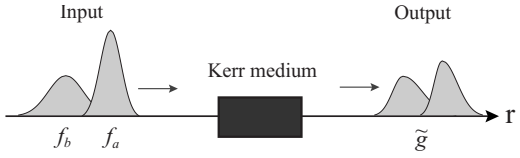

FIG. 1. Schematic illustration of the situation considered in this study. Two photons with different mode functions [$f_a(r)$ and $f_b(r)$] are input into a nonlinear optical system.

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$$\mathcal{V} = \int dr f_a(r) f_b^*(r). \tag{2}$$

$|\mathcal{V}| \leq 1$ by definition. $\tilde{c}_r$ is a photon annihilation operator in the real-space representation. [An operator without a tilde (c_k) is used to denote a photon annihilation operator in the wave number representation. $\tilde{c}_r$ and c_k are connected through the Fourier transformation as $\tilde{c}_r = (2\pi)^{-1/2} \int dk e^{ikr} c_k$.]

After interaction with the Kerr system, two photons are output having the same polarization. The output state vector $|AB_{\text{out}}\rangle$ is given by

$$|AB_{\text{out}}\rangle = 2^{-1/2} \int dr_1 dr_2 \tilde{g}(r_1, r_2) \tilde{c}_{r_1}^\dagger \tilde{c}_{r_2}^\dagger |0\rangle, \tag{3}$$

where the output wave function $\tilde{g}$ is symmetrized as $\tilde{g}(r_1, r_2) = \tilde{g}(r_2, r_1)$, and is normalized as $\int dr_1 dr_2 |\tilde{g}(r_1, r_2)|^2 = 1$. Due to the nonlinear interaction, the output wave function $\tilde{g}$ is not factorizable in general.

In order to evaluate the optical nonlinearity appearing in the output state of Eq. (3), we define the *linear* output state by

$$|AB_{\text{out}}^L\rangle = (1 + |\mathcal{V}|^2)^{-1/2} \int dr_1 dr_2 \bar{f}_a(r_1) \bar{f}_b(r_2) \tilde{c}_{r_1}^\dagger \tilde{c}_{r_2}^\dagger |0\rangle, \tag{4}$$

where $\bar{f}_a(r)$ and $\bar{f}_b(r)$ are the one-photon output wave functions, which are resultants of the single-photon inputs, such as $\int dr f_a(r) c_r^\dagger |0\rangle$. Note that $\mathcal{V}$ is conserved, namely, $\int dr \bar{f}_a(r) \bar{f}_b^*(r) = \int dr f_a(r) f_b^*(r) = \mathcal{V}$. The two-photon Kerr effect is quantified by the fidelity α between $|AB_{\text{out}}\rangle$ and $|AB_{\text{out}}^L\rangle$, as given by

$$\alpha = \left(\frac{2}{1 + |\mathcal{V}|^2} \right)^{1/2} \int dr_1 dr_2 \bar{f}_a^*(r_1) \bar{f}_b^*(r_2) \tilde{g}(r_1, r_2). \tag{5}$$

$|\alpha| \leq 1$ by definition. The nonlinear effect appears as the deviation of α from unity. When $|\alpha| = 1$ ($\alpha = e^{i\theta}$), θ represents the nonlinear phase shift, which is applicable to optical phase gates. However, in general, the nonlinear effect degrades the fidelity, namely, $|\alpha| < 1$. Thus, the guiding principle for designing an optical phase gate is to maximize the nonlinear phase shift θ while keeping $|\alpha|$ close to unity.

III. SEMICLASSICAL EVALUATION

Since a fully quantum-mechanical calculation of $\tilde{g}$ is difficult in general, we develop a semiclassical method to evaluate α. To this end, we consider the dynamics of a classical light pulse, the amplitude of which is given by $\mu f_a(r) + \nu f_b(r)$, where μ and ν are perturbation coefficients. The state vector corresponding to this classical pulse is given by

$$|\Phi_{\text{in}}\rangle = \mathcal{N} \exp\left(\int dr [\mu f_a(r) + \nu f_b(r)] \tilde{c}_r^\dagger \right) |0\rangle, \tag{6}$$

where the normalization constant $\mathcal{N}$ is given by $\mathcal{N} = \exp[-(|\mu|^2 + |\nu|^2 + \mu\nu^* \mathcal{V} + \mu^* \nu \mathcal{V}^*)/2]$. Up to the two-photon components, the above state vector can be expanded as

$$|\Phi_{\text{in}}\rangle = \mathcal{N}[|0\rangle + \mu |A_{\text{in}}\rangle + \nu |B_{\text{in}}\rangle + 2^{-1/2} \mu^2 |AA_{\text{in}}\rangle + 2^{-1/2} \nu^2 |BB_{\text{in}}\rangle + (1 + |\mathcal{V}|^2)^{1/2} \mu\nu |AB_{\text{in}}\rangle], \tag{7}$$

where $|A_{\text{in}}\rangle$ and $|AA_{\text{in}}\rangle$ are the one- and two-photon states for the f_a mode, respectively, and are given by $|A_{\text{in}}\rangle = \int dr f_a(r) \tilde{c}_r^\dagger |0\rangle$ and $|AA_{\text{in}}\rangle = 2^{-1/2} [\int dr f_a(r) \tilde{c}_r^\dagger]^2 |0\rangle$. The output state vector for $|\Phi_{\text{in}}\rangle$ is given, as a result of quantum time evolution, by $|\Phi_{\text{out}}\rangle = e^{-i\mathcal{H}(t-t_0)} |\Phi_{\text{in}}\rangle$, where $\mathcal{H}$ is the Hamiltonian for the overall system including both the photon field and the material system, t_0 denotes the initial time, and t should be chosen as an arbitrary time sufficiently long after the interaction. Due to the linearity in the quantum time evolution, $|\Phi_{\text{out}}\rangle$ is given by

$$|\Phi_{\text{out}}\rangle = \mathcal{N}[|0\rangle + \mu |A_{\text{out}}\rangle + \nu |B_{\text{out}}\rangle + 2^{-1/2} \mu^2 |AA_{\text{out}}\rangle + 2^{-1/2} \nu^2 |BB_{\text{out}}\rangle + (1 + |\mathcal{V}|^2)^{1/2} \mu\nu |AB_{\text{out}}\rangle]. \tag{8}$$

The output amplitude $\mathcal{F}(r)$ is given by $\mathcal{F}(r) = \langle \Phi_{\text{out}} | c_r | \Phi_{\text{out}} \rangle$. The components proportional to ν and $\mu\nu\mu^*$, denoted hereafter as $\mathcal{F}^{(\nu)}(r)$ and $\mathcal{F}^{(\mu\nu\mu^*)}(r)$, are given by

$$\mathcal{F}^{(\nu)}(r) = \bar{f}_b(r), \tag{9}$$

$$\mathcal{F}^{(\mu\nu\mu^*)}(r) = (2 + 2|\mathcal{V}|^2)^{1/2} \int dr' \bar{f}_a^*(r') \tilde{g}(r, r') - \mathcal{V}^* \bar{f}_a(r) - \bar{f}_b(r). \tag{10}$$

From Eqs. (9) and (10), it is readily confirmed that the quantity α' defined by

$$\alpha' = 1 + (1 + |\mathcal{V}|^2)^{-1} \int dr [\mathcal{F}^{(\nu)}(r)]^* \mathcal{F}^{(\mu\nu\mu^*)}(r) \tag{11}$$

becomes identical to the target quantity α defined in Eq. (5). Note that $\mathcal{F}^{(\nu)}(r)$ and $\mathcal{F}^{(\mu\nu\mu^*)}(r)$ are accessible within semiclassical theory. Thus, in terms of semiclassical results, the two-photon Kerr effect can be evaluated using Eq. (11), avoiding the need to perform fully quantum-mechanical calculations.

Three comments should be made regarding Eq. (11). (i) Evaluation of α is also possible in terms of $\mathcal{F}^{(\mu)}$ and $\mathcal{F}^{(\mu\nu\nu^*)}$. (ii) Although it is assumed throughout this study that the two input photons have the same polarization, Eq. (11) is also applicable to the case in which the two photons have different polarizations. Thus, Eq. (11) is applicable to any two input photons, regardless of their polarizations and the overlap $\mathcal{V}$. (iii) Derivation of Eq. (11) depends only on the linearity of quantum time evolution; thus, Eq. (11) is applicable to general Kerr systems in the dissipation-free limit.

IV. TRANSITIONAL BEHAVIOR BETWEEN THE SELF-KERR AND CROSS-KERR EFFECTS

A. System and input mode functions

In the following, we apply Eq. (11) to a model Kerr system and observe the transitional behavior between the self- and cross-Kerr effects by two photons. As an example of a

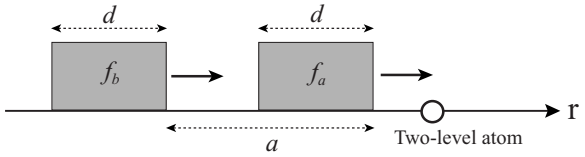

FIG. 2. Profile of rectangular input photons. Both photons have the same pulse length d and are resonant with the atom. a denotes the delay of the second photon.

Kerr system, we employ a one-dimensional atom [6,7]. It is realized in the weak coupling ($\kappa \gg g$) and dissipationless ($\gamma \to 0$) limit of a cavity quantum electrodynamics system, where κ is the cavity decay rate, g is the coupling between the cavity photon and the atom, and γ is the atomic emission rate into the noncavity photonic modes. Setting $\hbar = c = 1$, denoting the Pauli destruction operator of the atomic excitation by σ, and choosing the resonance of the atom as the origin of the energy, the Hamiltonian of the overall system including the atom and the photon field is given by

$$\mathcal{H} = \int dk[kc_k^\dagger c_k + \sqrt{\Gamma/2\pi}(c_k^\dagger \sigma + \sigma^\dagger c_k)], \tag{12}$$

where c_k is the photon annihilation operator in the wave number representation, and $\Gamma(=4g^2/\kappa)$ is the atomic decay rate into the cavity output [22]. Derivation of Eq. (12) from the conventional cavity-QED Hamiltonian is presented in Appendix A. Regarding the mode functions f_a and f_b of the input photons, the following rectangular forms are assumed (see Fig. 2):

$$f_a(r) = \begin{cases} d^{-1/2} & (-d < r < 0) \\ 0 & (\text{otherwise}), \end{cases} \tag{13}$$

$$f_b(r) = f_a(r + a), \tag{14}$$

where d denotes the pulse length and a denotes the delay. The input photons are assumed to be resonant with the atom. $\mathcal{V} = 1 - a/d$ when $a < d$, and $\mathcal{V} = 0$ when $a > d$. Namely, a complete self-Kerr situation is realized when $a = 0$, and transitional behavior is observed as a is increased.

B. Semiclassical evaluation of α

In order to carry out the semiclassical evaluation, we derive the semiclassical equations of motion from the Hamiltonian of Eq. (12) and the classical input $|\Phi_{\text{in}}\rangle$ given by Eq. (6). The Heisenberg equation for the atomic operator σ is given by (see Appendix B)

$$\frac{d}{dt}\sigma = -\frac{\Gamma}{2}\sigma - i\sqrt{\Gamma}(1 - 2\sigma^\dagger\sigma)\tilde{c}_{t_0 - t}(t_0), \tag{15}$$

where t_0 is the initial moment, and $\tilde{c}_{t_0 - t}(t_0)$ is the initial photon annihilation operator in the real-space representation, with the space coordinate $r = t_0 - t(<0)$ [23,24]. By taking the expectation value with $|\Phi_{\text{in}}\rangle$, the equation of motion for $\langle\sigma\rangle \equiv \langle\Phi_{\text{in}}|\sigma(t)|\Phi_{\text{in}}\rangle$ is given by

$$\frac{d}{dt}\langle\sigma\rangle = -\frac{\Gamma}{2}\langle\sigma\rangle - i\sqrt{\Gamma}(1 - 2\langle\sigma^\dagger\sigma\rangle)[\mu f_a(t_0 - t) + \nu f_b(t_0 - t)]. \tag{16}$$

Note that the initial field operator $\tilde{c}_r(t_0)$ is rigorously replaceable with a c-number amplitude $\mu f_a(r) + \nu f_b(r)$, since $|\Phi_{\text{in}}\rangle$ is in a coherent state. Thus, the equation of motion for $\langle\sigma\rangle$ is reduced to a semiclassical one. In this manner, the semiclassical equations for the five relevant quantities ($\langle\sigma^{(\mu)}\rangle$, $\langle\sigma^{(\nu)}\rangle$, $\langle\sigma^\dagger\sigma^{(\mu\mu^*)}\rangle$, $\langle\sigma^\dagger\sigma^{(\mu\nu^*)}\rangle$, and $\langle\sigma^{(\mu\nu\mu^*)}\rangle$) are derivable. Using the input-output relation, as given by (see Appendix B)

$$\tilde{c}_r(t) = \tilde{c}_{r - t + t_0}(t_0) - i\sqrt{\Gamma}\sigma(t - r), \tag{17}$$

the output amplitude $\mathcal{F}(r,t) \equiv \langle\tilde{c}_r(t)\rangle$ is given by

$$\mathcal{F}(r,t) = \mu f_a(r - t + t_0) + \nu f_b(r - t + t_0) - i\sqrt{\Gamma}\langle\sigma(t - r)\rangle. \tag{18}$$

Thus, the principal task in the semiclassical evaluation of α' is to solve the differential equations for five quantities. Using the dimensional lengths $\tilde{d}(\equiv\Gamma d)$ and $\tilde{a}(\equiv\Gamma a)$, α' is given for $a > d$ by

$$\alpha' = 1 - \frac{16}{3\tilde{d}^2}e^{\tilde{d} - \tilde{a}}(1 - e^{-\tilde{d}/2})^4(1 + 2e^{-\tilde{d}/2}), \tag{19}$$

and for $a < d$ by

$$\alpha' = 1 - \frac{16(I_1 + I_2 + I_3)}{\tilde{d}^2 + (\tilde{d} - \tilde{a})^2}, \tag{20}$$

where $I_1 = 2\int_{\tilde{a} - \tilde{d}}^0 dr(1 - e^{(r - \tilde{a})/2})^2(1 - e^{r/2})^2$, $I_2 = 2(1 - e^{-\tilde{d}/2})^2 \times \int_{-\tilde{a}}^0 dr(1 - e^{(r + \tilde{a} - \tilde{d})/2})^2 e^r$, and $I_3 = (1 - e^{-\tilde{d}/2})^4 e^{-\tilde{a}}$.

C. Quantum-mechanical evaluation

Usually, the fully quantum-mechanical calculation of the output wave function $\tilde{g}$ is difficult, even numerically. However, exceptionally for this simple system, the output wave functions ($\bar{f}_a$, $\bar{f}_b$, and $\bar{g}$) can be obtained analytically and α can be evaluated directly using Eq. (5) [22,25,26]. It is confirmed that α thus evaluated coincides with α' of Eqs. (19) and (20), proving the validity of the semiclassical method.

D. Results

In Fig. 3, α is plotted as a function of the delay a of the second pulse. Regarding the coherence length d of the photons, it is expected that large nonlinear effects can be obtained when d is of the order of Γ^{-1} [26]. In the present case of rectangular input pulses, it is confirmed numerically that α is maximized when $d \simeq 5.5\Gamma^{-1}$. Thus, d is fixed at $d = 5.5\Gamma^{-1}$ in Fig. 3, and at $d = \Gamma^{-1}$ for reference. When both input photons are resonant with the atom, α takes a real value as shown in Fig. 3, but generally α takes a complex value satisfying $|\alpha| \leq 1$.

As expected, the Kerr effect is strong when two photons are input almost simultaneously ($a \simeq 0$). As the delay is in-

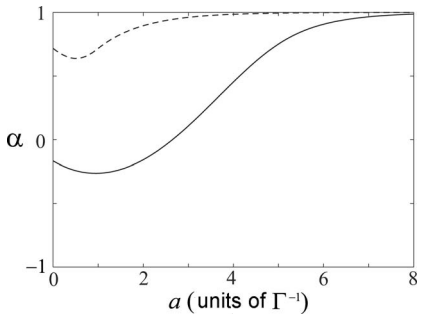

FIG. 3. Dependence of the two-photon Kerr effect α on the pulse delay a. The coherence length d of photons is chosen at $d=5.5\Gamma^{-1}$ (solid line) and at $d=\Gamma^{-1}$ (dashed line).

creased ($a\to\infty$), the Kerr effect becomes weaker ($|\alpha-1|\to 0$). A notable fact is that the Kerr effect is not maximized for the completely self-Kerr case of $a=0$. As shown in Fig. 3, the Kerr effect is maximized when the second photon is slightly delayed. Since the origin of the Kerr effect in the two-level atom is absorption saturation, this optimum delay time can be understood as the time required for absorption of the first photon by the atom. Similar phenomena are widely observed in four-wave mixing experiments using intense laser fields.

For rectangular input pulses, the two-photon Kerr effect is maximized when $d=5.57\Gamma^{-1}$ and $a=0.96\Gamma^{-1}$, where $\alpha=-0.26$. On the other hand, when Gaussian pulses are used as the input, $\alpha=-0.47$ can be attained even when just considering the self-Kerr ($a=0$) case. Thus, Gaussian pulses are more advantageous than rectangular pulses for maximizing the Kerr effect. This demonstrates that the two-photon Kerr effect is sensitive not only to the pulse length d and the delay a but also to the profiles of the input mode functions, f_a and f_b. Determination of the optimum mode functions for maximizing the two-photon Kerr effect is left as a future problem.

V. SUMMARY

The achievements of this study are summarized in the following two points. (i) We have developed a semiclassical method for evaluating the Kerr effect between two photons with arbitrary mode functions f_a and f_b and with arbitrary polarizations. The method comprises the following two procedures: assuming that a classical light pulse (with amplitude $\mu f_a+\nu f_b$) is input, the linear and third-order components of the output amplitude are calculated using semiclassical theory; then, the two-photon Kerr effect is determined using Eq. (11). This method drastically reduces computation by replacing the fully quantum-mechanical calculation with a semiclassical one. (ii) Choosing a two-level atom as a nonlinear agent and two rectangular photon pulses as the input (see Fig. 2), the transitional behavior between the self-Kerr and cross-Kerr effects was investigated. The semiclassical method enables us to evaluate the Kerr effect by solving the differential equations for only five quantities. It was revealed that the maximum Kerr effect is obtained when there is a slight time delay between the two photons, namely, the intermediate situations ($0<|\mathcal{V}|<1$) of the self-Kerr and cross-Kerr cases. Thus, the optimum design of two-photon nonlinear devices is likely to be found in this intermediate region. Since the semiclassical evaluation method presented here is applicable to the intermediate situation, this method will serve as the best approach for these cases.

ACKNOWLEDGMENTS

The author is grateful to N. Matsuda, K. Edamatsu, A. Ishikawa, H. Ishihara, and Y. Shinozuka for fruitful discussions. This research was partially supported by the Research Foundation for Opto-Science and Technology, by CREST project of JST, and by a Grant-in-Aid for Creative Scientific Research (Grant No. 17GS1204).

APPENDIX A: DERIVATION OF EQ. (12) FROM THE CONVENTIONAL CAVITY-QED HAMILTONIAN

Here, we derive the Hamiltonian of a one-dimensional atom [Eq. (12)] from the conventional cavity-QED Hamiltonian. A cavity-QED system composed of a single two-level atom and a one-sided cavity is characterized by the following three parameters: g (the coupling between the cavity photon and the atom), κ (the cavity decay rate), and γ (the atomic emission rate into the noncavity photonic modes). Assuming that the atomic transition frequency and the cavity resonance frequency are identical and taking them as the origin of energy, the Hamiltonian is given by

$$\mathcal{H}=g(\sigma^\dagger c+c^\dagger\sigma)+\int dk[kb_k^\dagger b_k+\sqrt{\kappa/2\pi}(c^\dagger b_k+b_k^\dagger c)]$$
$$+\int dk[kd_k^\dagger d_k+\sqrt{\gamma/2\pi}(\sigma^\dagger d_k+d_k^\dagger\sigma)], \tag{A1}$$

where c, b_k, and d_k are the photon annihilation operators for the cavity mode, the input-output field of cavity with wave number k, and the noncavity mode with wave number k, respectively, and σ is the Pauli destruction operator for the atomic excitation. Note that the photonic operators are orthonormalized as $[c,c^\dagger]=1$ and $[b_k,b_{k'}^\dagger]=[d_k,d_{k'}^\dagger]=\delta(k-k')$.

By diagonalizing the second term, c and b_k are recast into a single continuum, denoted by c_k. It is given by

$$c_k=\xi_1(k)c+\int dq\xi_2(k,q)b_q, \tag{A2}$$

$$\xi_1(k)=\frac{(2/\pi\kappa)^{1/2}}{1-2ik/\kappa}, \tag{A3}$$

$$\xi_2(k,q)=\frac{\pi^{-1}}{(1-2ik/\kappa)(k-q+i\delta)}+i\delta(k-q), \tag{A4}$$

which is orthonormalized as $[c_k,c_{k'}^\dagger]=\delta(k-k')$. Using that $c=\int dk\xi_1^*(k)c_k$, and taking the $\gamma\to 0$ limit (by which d_k is decoupled from the atom), the Hamiltonian is recast into the following form:

$$\mathcal{H} = \int dk\{kc_k^\dagger c_k + [g\xi_1^*(k)\sigma^\dagger c_k + \text{H.c.}]\}. \tag{A5}$$

The coupling $g\xi_1^*(k)$ between σ and c_k is dependent on the photonic energy k. However, when κ is large and nearly resonant photons (satisfying $|k| \ll \kappa$) are concerned, one may neglect this k dependence and set $g\xi_1^*(k) = \sqrt{\Gamma/2\pi}$, where $\Gamma = 4g^2/\kappa$. Thus, the Hamiltonian of a one-dimensional atom [Eq. (12)] has been derived.

APPENDIX B: DERIVATION OF THE INPUT-OUTPUT RELATION

The Heisenberg equation for c_k is given from Eq. (12) by

$$\dot{c}_k = -ikc_k - i\sqrt{\Gamma/2\pi}\sigma. \tag{B1}$$

Denoting the initial and final moments by t_0 and t_1, the operator at time τ $(t_0 < \tau < t_1)$ is represented in two ways:

$$c_k(\tau) = c_k(t_0)e^{-ik(\tau-t_0)} - i\sqrt{\frac{\Gamma}{2\pi}}\int_{t_0}^{\tau} d\tau' \sigma(\tau')e^{-ik(\tau-\tau')} \tag{B2}$$

$$= c_k(t_1)e^{-ik(\tau-t_1)} + i\sqrt{\frac{\Gamma}{2\pi}}\int_{\tau}^{t_1} d\tau' \sigma(\tau')e^{-ik(\tau-\tau')}. \tag{B3}$$

Using the above two forms of $c_k(\tau)$, $\int dk c_k(\tau)$ is recast into the following two forms:

$$\int dk c_k(\tau) = \sqrt{2\pi}\tilde{c}_{t_0-\tau}(t_0) - i\sqrt{\frac{\pi\Gamma}{2}}\sigma(\tau) \tag{B4}$$

$$= \sqrt{2\pi}\tilde{c}_{t_1-\tau}(t_1) + i\sqrt{\frac{\pi\Gamma}{2}}\sigma(\tau). \tag{B5}$$

Note that $\tilde{c}_r$ is in the real-space representation, as defined by $\tilde{c}_r = (2\pi)^{-1/2}\int dk e^{ikr}c_k$. Equating the right-hand sides and introducing new labels $r(=t_1-\tau)$ and $t(=t_1)$, we obtain the input-output relation [Eq. (17)], in which the output field operator at time t is expressed in terms of the input field operator at t_0 and the atomic operator at $t-r$. The Heisenberg equation for the atomic operator σ [Eq. (15)] is obtained from Eqs. (12) and (B4).

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