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Squeezing in the output field from a one-dimensional atom

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Abstract
It is revealed that quadrature squeezing occurs in the output field from a one-dimensional atom driven by a classical field. The degree of squeezing depends on the intensity of the input field, and reaches 28% at the maximum. It can roughly be regarded that the output field is in a superposition of coherent and number states.

1. Introduction

In the field of cavity quantum electrodynamics (QED), highly isolated quantum systems composed of cavity photons and atoms can be realized [1]. The principal merit of a cavity QED system lies in its long coherence time. Such good quantum coherence has enabled us to observe experimentally the quantum oscillation among several quantum levels [2, 3]. This oscillatory dynamics has been applied to the preparation of non-classical states among cavity photons and atoms [4, 5], and also to quantum-state measurements [6–8]. The potential of cavity QED systems in quantum state engineering is further enriched with the help of laser-induced coherence among the atomic levels [9]. For example, the generation of single-photons with controllable pulse shapes has been accomplished [10, 11]. Recently, it was pointed out that the generation of entangled photons is also possible in cavity QED systems by utilizing the motional degrees of freedom of an atom [12, 13].

Besides isolation of the internal atom–photon system, the cavity also acts as an amplifier of the photon field. Thus, a cavity QED system is also promising as a giant $\chi^{(3)}$ system, in which the atomic Kerr nonlinearity due to transition saturation is drastically enhanced by a cavity. One demonstration of this giant nonlinear effect is the photon blockade, in which the cavity photon mode behaves as an effective two-level system with transition saturation [14–16]. Another demonstration of the giant $\chi^{(3)}$ effect is a significant nonlinear phase shift obtained by extremely weak input fields [17, 18]. The experimental results suggest realization of the strong optical nonlinearity sensitive to individual photons [19], which can be applicable to the construction of the controlled-phase gates required for optical quantum computing [20–24].

In this experiment, classical fields were used as the input, and the observed phase shift was quantitatively explained through semiclassical analysis, in which the photon fields are described solely by their amplitudes [18]. The semiclassical theories predict that the amplitude of the output field becomes smaller than that of the input, even in the dissipationless case in which the field energy must be conserved between the input and the output (see section 3) [25, 26]. This fact implies that the output field is not in a simple coherent state, and stimulates us to investigate the field fluctuation in the output field. In this study, it is revealed that the output field may exhibit squeezing in one quadrature, when the input intensity is weak. This is another manifestation of the weak-field optical nonlinearity peculiar to a cavity QED system besides the nonlinear phase shift.

This study is presented as follows. In section 2, the Hamiltonian for a cavity QED system in the region called a *one-dimensional* atom is presented, and the basic equations are derived. In section 3, the mean amplitude of the output field is calculated based on the semiclassical equations. Section 4 is the main part of this study, where the squeezing spectra are calculated as functions of the input intensity, revealing the optimum intensity and the maximum degree of squeezing. It is also shown that the squeezing spectra can be explained qualitatively by a superposition of coherent and number states.

2. Model and basic equations

The system considered in this study is composed of a one-sided cavity and a two-level atom placed inside of it. An atom confined in an optical cavity is coupled to two kinds of photonic continua: the quasi-cavity continuum and the noncavity continuum [27]. When the coupling to the former dominates that to the latter (in the language of cavity QED, $g^2/\kappa \gg \gamma$) and in the weak coupling regime ($\kappa \gg g$), the atom behaves as if it were coupled only to a one-dimensional photon field of the quasi-cavity continuum and is called a one-dimensional atom [17, 18]. In the lossless limit ($\gamma \to 0$), the Hamiltonian describing this system is given by

$$\mathcal{H} = \int \mathrm{d}\omega \left[\omega b_\omega^\dagger b_\omega + \sqrt{\frac{\Gamma}{2\pi}} \left(\mathrm{i} b_\omega^\dagger s + \mathrm{H.c.} \right) \right], \tag{1}$$

where s and b_ω represent the annihilation operators for the atomic excitation and an external photonic mode with frequency ω, respectively. The commutators for s and b_ω are given by $[s, s^\dagger] = 1 - 2s^\dagger s$ and $\left[b_\omega, b_{\omega'}^\dagger\right] = \delta(\omega - \omega')$. The atomic decay rate Γ is represented, in terms of the cavity QED parameters, as $\Gamma = 4g^2/\kappa$. Note that $\hbar = c = 1$ throughout this study, and that the free Hamiltonian for the atom ($\omega_a s^\dagger s$) has been removed by choosing ω_a as the origin of the energy.

On the basis of this Hamiltonian, we investigate the optical response of a one-dimensional atom using the Heisenberg equations. Following the standard input–output formalism [28–31], the input and output field operators are defined by

$$a_{\mathrm{in}}[t] = -(2\pi)^{-1/2} \int_{-\infty}^{\infty} \mathrm{d}\omega \, \mathrm{e}^{-\mathrm{i}\omega(t-t_0)} b_\omega(t_0), \tag{2}$$

$$a_{\mathrm{out}}[t] = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \mathrm{d}\omega \, \mathrm{e}^{-\mathrm{i}\omega(t-t_1)} b_\omega(t_1), \tag{3}$$

where t_0 and t_1 represent the initial and final moments, and $t_0 < t < t_1$. Note that square brackets are used in the definitions of $a_{\mathrm{in}}[t]$ and $a_{\mathrm{out}}[t]$, since these operators are not in the Heisenberg representation. $a_{\mathrm{in}}[t]$ and $a_{\mathrm{out}}[t]$ are connected through the following boundary condition:

$$a_{\mathrm{in}}[t] + a_{\mathrm{out}}[t] = \sqrt{\Gamma} s(t). \tag{4}$$

Using the input field operator $a_{\mathrm{in}}[t]$, the Heisenberg equation for s is given by

$$\frac{\mathrm{d}}{\mathrm{d}t} s = -\frac{\Gamma}{2} s + \sqrt{\Gamma}(1 - 2s^\dagger s) a_{\mathrm{in}}[t]. \tag{5}$$

3. Semiclassical theory

Before investigating the quantum fluctuation in the output field, we first examine the mean amplitude of the output field. Throughout this study, we concentrate on a case where the input field is a monochromatic coherent field with amplitude E and frequency $\tilde{\Omega}$. Hereafter, we principally use a dimensionless frequency $\Omega (= \tilde{\Omega}/\Gamma)$. In this case, one can rigorously replace the operator $a_{\mathrm{in}}[t]$ with a c-number $\langle a_{\mathrm{in}} \rangle = E\, \mathrm{e}^{-\mathrm{i}\Omega\Gamma(t-t_0)}$ when $a_{\mathrm{in}}[t]$ operates on the initial state vector $|i\rangle$. Using this fact and equation (5), and denoting the expectation value $\langle i|A(t)|i\rangle$ by $\langle A\rangle$, the equations of motion for $\langle s\rangle$ and $\langle s^\dagger s\rangle$ are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle s\rangle = -\frac{\Gamma}{2}\langle s\rangle + \sqrt{\Gamma}(1 - 2\langle s^\dagger s\rangle)\langle a_{\mathrm{in}}\rangle, \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle s^\dagger s\rangle = -\Gamma\langle s^\dagger s\rangle + \sqrt{\Gamma}(\langle a_{\mathrm{in}}\rangle\langle s\rangle^* + \mathrm{c.c.}). \tag{7}$$

These equations are nothing but the semiclassical Bloch equations for an atom driven by a classical field. The stationary solutions of the above equations are readily obtained as

$$\langle s\rangle = \frac{2(1 + 2\mathrm{i}\Omega)}{|1 + 2\mathrm{i}\Omega|^2 + J}\frac{\langle a_{\mathrm{in}}\rangle}{\sqrt{\Gamma}}, \tag{8}$$

$$\langle s^\dagger s\rangle = \frac{J/2}{|1 + 2\mathrm{i}\Omega|^2 + J}, \tag{9}$$

where J denotes the dimensionless intensity of the input field, defined by $J = 8|\langle a_{\mathrm{in}}\rangle|^2/\Gamma$. The amplitude $\langle a_{\mathrm{out}}\rangle$ of the output field is given, using equations (4) and (8), by

$$\langle a_{\mathrm{out}}\rangle = \frac{(1 + 2\mathrm{i}\Omega)^2 - J}{|1 + 2\mathrm{i}\Omega|^2 + J}\langle a_{\mathrm{in}}\rangle. \tag{10}$$

One can readily confirm that the amplitude of the output field is attenuated, i.e., $|\langle a_{\mathrm{out}}\rangle| < |\langle a_{\mathrm{in}}\rangle|$ [25, 26]. The field amplitude is conserved only in the linear limit, $J \to 0$. Therefore, this attenuation in the mean amplitude should be regarded as a nonlinear effect. In particular, $\langle a_{\mathrm{out}}\rangle$ completely vanishes when the input field is resonant with the atom ($\Omega = 0$) and the intensity $J = 1$. It might appear that the energy conservation is broken, in spite of the fact that there is no dissipation mechanism in the present system. However, by evaluating the field intensity $\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}}\rangle$ using equations (4), (8) and (9), one can confirm the following energy conservation law:

$$\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}}\rangle = \langle a_{\mathrm{in}}^\dagger a_{\mathrm{in}}\rangle = |\langle a_{\mathrm{in}}\rangle|^2. \tag{11}$$

These observations reveal that the inequality $\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}}\rangle > |\langle a_{\mathrm{out}}\rangle|^2$ holds in the output field. This inequality suggests that the output field is not in a genuine coherent state, and therefore cannot be characterized solely by the mean amplitude. This fact motivates us to examine the field fluctuation in the next section.

4. Spectrum of squeezing

In this section, we examine the variances in the quadratures of the output field. The quadrature operators $X_{1,2}$ for the output field are defined by

$$X_1^{\mathrm{out}}[t] = \mathrm{e}^{\mathrm{i}\theta} a_{\mathrm{out}}^{\dagger}[t] + \mathrm{e}^{-\mathrm{i}\theta} a_{\mathrm{out}}[t], \tag{12}$$

$$X_2^{\mathrm{out}}[t] = \mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta} a_{\mathrm{out}}^{\dagger}[t] - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\theta} a_{\mathrm{out}}[t], \tag{13}$$

where θ denotes the reference phase. The quantities we evaluate here are the squeezing spectra: $S_j^{\mathrm{out}}(\omega)$: $(j = 1, 2)$, which is defined as the Fourier transform of the normally ordered two-time correlation function $\langle : X_j^{\mathrm{out}}[t], X_j^{\mathrm{out}}[0] : \rangle$:

$$: S_j^{\mathrm{out}}(\omega) := \int \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}\omega\Gamma t} \left\langle : X_j^{\mathrm{out}}[t], X_j^{\mathrm{out}}[0] : \right\rangle, \tag{14}$$

where $\langle A, B\rangle = \langle AB\rangle - \langle A\rangle\langle B\rangle$ [30]. Note that ω is a dimensionless frequency normalized by Γ. The negativity of $: S_j^{\mathrm{out}} :$ implies squeezing in the X_j quadrature. The two-time correlation functions for the quadrature operators are related to those for the atomic operators by

$$\left\langle : X_j^{\mathrm{out}}[t], X_j^{\mathrm{out}}[0] : \right\rangle = \Gamma[\langle s^{\dagger}(t), s(0)\rangle + (-1)^j\, \mathrm{e}^{-2\mathrm{i}\theta} T\langle s(t), s(0)\rangle] + \mathrm{c.c.}, \tag{15}$$

where T denotes the time ordering.

Now we start to evaluate the two-time correlation functions, $x(t) = \langle s^{\dagger}(t), s(0)\rangle$ and $y(t) = \mathrm{e}^{-2\mathrm{i}\theta} T\langle s(t), s(0)\rangle$. Denoting $z(t) = \mathrm{e}^{-\mathrm{i}\theta}\langle s^{\dagger}(t)s(t), s(0)\rangle$ and $\langle \tilde{a}_{\mathrm{in}}\rangle = \mathrm{e}^{-\mathrm{i}\theta}\langle a_{\mathrm{in}}\rangle$, we obtain from equation (5) the following equations of motion for the two-time correlation functions for $t > 0$:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\Gamma}{2}x - 2\sqrt{\Gamma}\langle \tilde{a}_{\mathrm{in}}\rangle^* z, \tag{16}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{\Gamma}{2}y - 2\sqrt{\Gamma}\langle \tilde{a}_{\mathrm{in}}\rangle z, \tag{17}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\Gamma z + \sqrt{\Gamma}(\langle \tilde{a}_{\mathrm{in}}\rangle x + \langle \tilde{a}_{\mathrm{in}}\rangle^* y). \tag{18}$$

Note that, in deriving equations (17) and (18), we have used the fact that $a_{\mathrm{in}}[t]$ and $s(0)$ are commutable if $t > 0$ [30]. The initial conditions are given by $x(0) = \langle s^{\dagger}s\rangle - |\langle s\rangle|^2$, $y(0) = -\mathrm{e}^{-2\mathrm{i}\theta}\langle s\rangle^2$, and $z(0) = -\mathrm{e}^{-\mathrm{i}\theta}\langle s\rangle\langle s^{\dagger}s\rangle$, where $\langle s\rangle$ and $\langle s^{\dagger}s\rangle$ are stationary values given by equations (8) and (9). The two-time correlation functions for negative t can be evaluated with the use of the following relations, $x(-|t|) = x^*(|t|)$ and $y(-|t|) = \mathrm{e}^{2\mathrm{i}\Omega|t|}y(|t|)$, which hold in the stationary state.

We hereafter focus on the case where the input field is resonant with the atom, i.e., $\Omega = 0$. In this case, $\langle a_{\mathrm{in}}\rangle$ becomes independent of time. By choosing the reference angle θ to satisfy $\mathrm{e}^{\mathrm{i}\theta} = \langle a_{\mathrm{in}}\rangle / |\langle a_{\mathrm{in}}\rangle|$, x, y and z are reduced to real quantities. Then, using the dimensionless intensity J, the two-time correlation functions are given by

$$x(t) + y(t) = C_1\, \mathrm{e}^{-\alpha_1\Gamma|t|} + C_2\, \mathrm{e}^{-\alpha_2\Gamma|t|}, \tag{19}$$

$$x(t) - y(t) = \frac{J}{2(1+J)}\, \mathrm{e}^{-\Gamma|t|/2}, \tag{20}$$

where α_1 and α_2 are the solutions of the quadratic equation $\alpha^2 - (3/2)\alpha + (1+J)/2 = 0$. C_1 and C_2 are given by

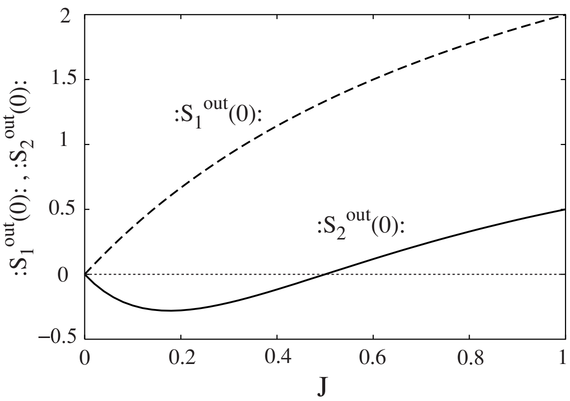

Figure 1. Plot of $:S_1^{\mathrm{out}}(0):$ and $:S_2^{\mathrm{out}}(0):$ as functions of the input intensity J. The thin dotted line represents $:S:=0$. When the input intensity is weak enough to satisfy $J < 1/2$, squeezing takes place in the X_2 quadrature.

$$C_1 = \frac{(\alpha_2 + 1/2)x(0) + (\alpha_2 - 1/2)y(0)}{\alpha_2 - \alpha_1}, \tag{21}$$

$$C_2 = \frac{(\alpha_1 + 1/2)x(0) + (\alpha_1 - 1/2)y(0)}{\alpha_1 - \alpha_2}, \tag{22}$$

where $x(0) = J^2/2(1+J)^2$ and $y(0) = -J/2(1+J)^2$. Using equations (14), (15), (19) and (20), the squeezing spectra are given by

$$:S_1^{\mathrm{out}}(\omega): = \frac{4J}{(1+J)|1+2\mathrm{i}\omega|^2}, \tag{23}$$

$$:S_2^{\mathrm{out}}(\omega): = \frac{4\alpha_1 C_1}{|\alpha_1 + \mathrm{i}\omega|^2} + \frac{4\alpha_2 C_2}{|\alpha_2 + \mathrm{i}\omega|^2}. \tag{24}$$

The squeezing spectra for the $\omega = 0$ component take particularly simple forms, which are given by

$$:S_1^{\mathrm{out}}(0): = \frac{4J}{(1+J)}, \tag{25}$$

$$:S_2^{\mathrm{out}}(0): = -\frac{4J(1-2J)}{(1+J)^3}. \tag{26}$$

The intensity dependences of $:S_j^{\mathrm{out}}(0):$ are plotted in figure 1. It is shown that the squeezing occurs in the X_2 quadrature when the input field is weak enough to satisfy $J < 1/2$. The squeezing is maximized when $J = (3-\sqrt{7})/2 = 0.177$, where the degree of squeezing reaches 28%. ($:S_2^{\mathrm{out}}(0): = -0.28$, in other words, $S_2^{\mathrm{out}}(0) = 0.72$.)

The intensity dependences of $:S_1^{\mathrm{out}}(0):$ and $:S_2^{\mathrm{out}}(0):$ observed in figure 1 can be intuitively understood as follows. The output field is expected to have two components, namely the coherent component and the incoherent component [25, 26, 32]. The coherent component is in a coherent state $|\alpha\rangle$ with a mean amplitude α. On the other hand, the incoherent component

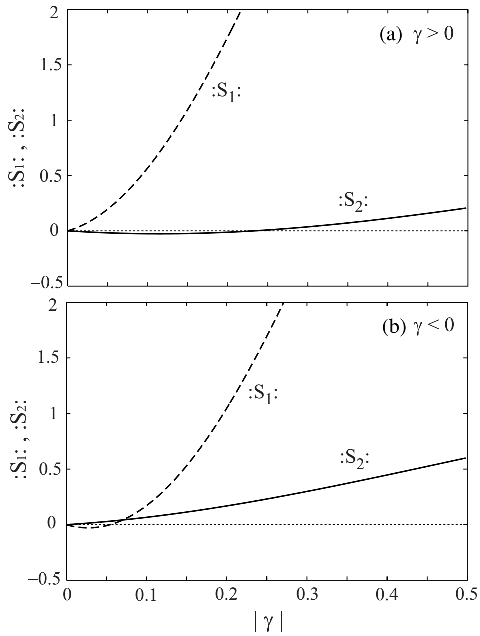

Figure 2. The normally ordered variances $:S_1:$ and $:S_2:$ for the state given by equation (27), as functions of the superposition coefficient $|\gamma|$. α is fixed at 3. γ is in phase with α (namely, γ is real and positive) in figure 2(a), whereas γ is out of phase with α in figure 2(b).

originates in the spontaneous emission from the atom, and therefore is in a number (single-photon) state $|1\rangle$. The quantum state of the output field would roughly be given by superposing these two components as

$$|\psi_{\mathrm{out}}\rangle \simeq C(|\alpha\rangle + \gamma|1\rangle), \tag{27}$$

where C is a normalization factor and γ is a superposition coefficient. Since the incoherent component originates in the spontaneous emission from the atom, $|\gamma|$ would be roughly proportional to the atomic excitation $\langle s^\dagger s\rangle$ and therefore to the input intensity J. The normally ordered variances $: S_1 :$ and $: S_2 :$ for this state are plotted in figure 2 as functions of the superposition coefficient $|\gamma|$. γ is in phase with α in figure 2(a), whereas γ is out of phase with α by π in figure 2(b). It is commonly observed in figures 2(a) and (b) that, as the incoherent component $|\gamma|$ increases, $: S_1 :$ is enhanced significantly whereas $: S_2 :$ is not so affected. This point is in agreement with figure 1. The phase of γ plays a crucial role in the small $|\gamma|$ region. When γ is in phase with α, as shown in figure 2(a), $: S_2:$ shows squeezing in the small $|\gamma|$ region, whereas $: S_1:$ is a monotonically increasing function of $|\gamma|$. In contrast, when γ is out of phase with α, as shown in figure 2(b), $: S_1 :$ shows squeezing in the very small $|\gamma|$ region whereas $: S_2:$ does not. Thus, the qualitative features of the quantum noise in the output port can be explained in terms of the wavefunction given by equation (27) with the coefficient γ in phase with α.

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