Quantum Anti-Zeno Effect by False Measurements

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We have investigated how the decay dynamics of an unstable quantum system is affected by a false measurement, where the decay is monitored by detecting a decay product but the active energy band of the detector does not match the energy of the decay product. It is shown that, although such a measurement is ineffective and has almost no effect if the detector response is slow, the detectability of decay is increased and the decay is accelerated considerably if the response is fast. This is due to the new decay channel, which is generated as a counteraction of measurement.

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It was predicted in 1977 that decay of an unstable quantum system is slowed down by frequently repeated measurements to confirm survival of the system, which is called the quantum Zeno effect [1]. Later, it was also pointed out that repeated measurements may result in acceleration of decay in some cases, which is called the quantum anti-Zeno effect [2,3]. The key assumption in these predictions was the projection postulate of quantum measurements [4]. Besides the Zeno and anti-Zeno effects, interesting possibilities of repeated measurements are revealed based on the projection postulate [5–7]. However, the postulate is applicable only to ideally performed measurements. In order to discuss more complicated measurement processes, one should analyze an enlarged quantum system, which includes, besides the target system to be measured, a part of the measurement apparatus [8,9]. In this formulation, decoherence due to measurements is naturally introduced and the postulate is no more required. Such a rigorous approach is particularly suitable for analyzing continuous measurements [10–15]. It was shown that the conventional theories of the Zeno and anti-Zeno effects based on the projection postulate can be reproduced from the rigorous formulation by regarding the response time of the apparatus as the measurement interval of discrete measurements, and by using an ideal measurement apparatus [11,14].

In the preceding discussions on the Zeno effects, it was usually assumed that measurement apparatus can detect the decay with high efficiency, because the Zeno effect is supposed to appear only weakly if measurements on the target system are ineffective [14]. Here, we investigate the effects of *false* measurements, where measurement apparatus seem to have no interaction with the target system and, therefore, to be unable to watch what is going on the target system. More concretely, we study a situation where the decay of a system is monitored by detecting a decay product emitted from the system, but the detection energy band of the apparatus does not match the energy of the decay product. We are showing that, even by such false measurements, one can detect the decay with a considerable probability if the response of the detector is quick, and that the decay rate is significantly enhanced, i.e., the anti-Zeno effect takes place. The mechanism and the condition for this phenomenon are also clarified.

As an unstable quantum system, we consider an excited atom which decays to the ground state accompanying an emission of a photon. The Hamiltonian of this system reads

$$\mathcal{H}_s = \Omega|x\rangle\langle x| + \int d\boldsymbol{k}[(\xi_{\boldsymbol{k}}|x\rangle\langle g|b_{\boldsymbol{k}} + \text{H.c.}) + k b_{\boldsymbol{k}}^\dagger b_{\boldsymbol{k}}], \tag{1}$$

where $|x\rangle$ and $|g\rangle$ are the excited and ground states of the atom, Ω is the atomic transition energy, and $b_{\boldsymbol{k}}$ is the annihilation operator for the photon with wave vector $\boldsymbol{k}$ (whose energy is $k = |\boldsymbol{k}|$), which is orthonormalized as $[b_{\boldsymbol{k}}, b_{\boldsymbol{k}'}^\dagger] = \delta(\boldsymbol{k} - \boldsymbol{k}')$. We hereafter restrict ourselves to a case where the atom-photon coupling $\xi_{\boldsymbol{q}}$ satisfies

$$\int d\boldsymbol{q}|\xi_{\boldsymbol{q}}|^2\delta(|\boldsymbol{q}| - k) = \frac{\gamma}{2\pi}. \tag{2}$$

In this case, the decay of the atom exactly follows the exponential law with the decay rate γ, and the line shape of the emitted photon is an exact Lorentzian with the central frequency Ω and the width $\gamma/2$ [16].

In the photodetection process, photons are converted to excitations (electron-hole pairs) in the detector. This process is modeled by the following photon-detector interaction term [17,18]:

$$\mathcal{H}_{sd} = \iint d\boldsymbol{k}\, d\omega\left[\left(\sqrt{\frac{\eta_k}{2\pi}}b_{\boldsymbol{k}}^\dagger c_{\boldsymbol{k}\omega} + \text{H.c.}\right) + \omega c_{\boldsymbol{k}\omega}^\dagger c_{\boldsymbol{k}\omega}\right], \tag{3}$$

where $c_{\boldsymbol{k}\omega}$ is the annihilation operator for the excitation in the detector with the total momentum $\boldsymbol{k}$ and energy ω, which is orthonormalized as $[c_{\boldsymbol{k}\omega}, c_{\boldsymbol{k}'\omega'}^\dagger] = \delta(\boldsymbol{k} - \boldsymbol{k}')\delta(\omega - \omega')$. Through this photon-detector interaction, the photon $\boldsymbol{k}$ is absorbed and counted by the detector with the rate $\eta_{\boldsymbol{k}}$. In order to discuss a false

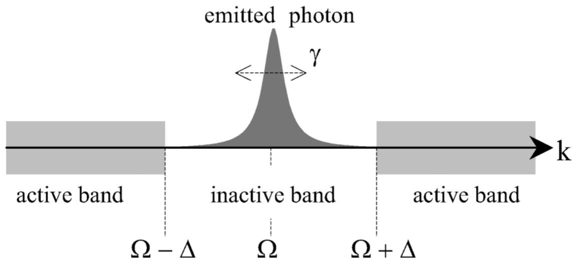

FIG. 1. Relation between the line shape of emitted photon and the active (inactive) band of the detector. When $\Delta \gg \gamma$, the emitted photon falls in the inactive band of the detector ($|k - \Omega| \leq \Delta$) almost completely.

measurement, we assume that the detector has an inactive energy band around the atomic transition energy Ω, which is realized by taking

$$\eta_{\boldsymbol{k}} = \eta_k = \begin{cases} \eta & (|k-\Omega| > \Delta) \\ 0 & (|k-\Omega| \leq \Delta). \end{cases} \quad (4)$$

Thus, while the detector does nothing on photons lying in the inactive band ($|k - \Omega| \leq \Delta$), the detector counts photons in the active band ($|k - \Omega| > \Delta$) with a response time $\tau \sim \eta^{-1}$. It is known that the response time τ plays an equivalent role to the measurement interval in discrete measurements [11]. Because the linewidth of the emitted photon is $\gamma/2$, the detector seems to be unable to catch the photon if $\Delta \gg \gamma$ (see Fig. 1). In fact, noting that the line shape is an exact Lorentzian, the fraction of photons emitted in the active band is given by $1 - (2/\pi)\arctan(2\Delta/\gamma)$. This amounts to only 3.2% for $\Delta = 10\gamma$, for example. Therefore, it is expected that almost no photons are counted by the detector and that such a false measurement does not affect the decay dynamics of the atom significantly.

In order to check the influence of such a false measurement, we see the temporal evolution of the system by the enlarged Hamiltonian $\mathcal{H} = \mathcal{H}_s + \mathcal{H}_{sd}$. Putting $|\psi(t)\rangle = e^{-i\mathcal{H}t}|x, 0, 0\rangle = f(t)|x, 0, 0\rangle + \int dk f_{\boldsymbol{k}}(t)|g, \boldsymbol{k}, 0\rangle + \iint dk\, d\omega\, f_{\boldsymbol{k}\omega}(t)|g, 0, \boldsymbol{k}\omega\rangle$, we focus on the following two probabilities; $1 - s(t) = 1 - |f(t)|^2 = \int dk |f_{\boldsymbol{k}}(t)|^2 + \iint dk\, d\omega |f_{\boldsymbol{k}\omega}(t)|^2$ (probability that the atom has decayed and has emitted a photon) and $r(t) = \iint dk\, d\omega |f_{\boldsymbol{k}\omega}(t)|^2$ (probability that the emitted photon is absorbed). The latter may be regarded as the probability of getting a detector response. It should be reminded that an exact decay law, $s(t) = e^{-\gamma t}$, is obtained when the system is not observed ($\eta = 0$). The temporal behaviors of $1 - s(t)$ and $r(t)$ are drawn in Fig. 2, where the inactive bandwidth $\Delta(= 10\gamma)$ is much larger than γ. When the detector response is slow ($\eta = \gamma$, solid curves in Fig. 2), the decay probability is almost unchanged from that of the unobserved case; i.e., $1 - s(t) \simeq 1 - e^{-\gamma t}$. Although a photon is emitted upon decay, detection of the emitted photon is almost unsuccessful; i.e., $r(t) \simeq 0$. Such behaviors of $1 - s(t)$ and $r(t)$ agree with our expectation on false measurements. However, when the detector response is fast ($\eta = 30\gamma$, broken curves in Fig. 2), we can observe that the detection probability of the emitted photon becomes surprisingly large ($\sim 40\%$). Furthermore, the decay is significantly promoted, which is nothing but the quantum anti-Zeno effect. In Fig. 3, $\ln s(t)/\gamma t$ is plotted to visualize the decay rate. The figure clarifies that the decay rate changes from the unobserved rate γ to the enhanced rate $\bar{\gamma}$ ($\simeq 1.6\gamma$) at $t \sim 0.1\gamma^{-1}$. Because the atom is kept almost excited at that moment, it decays with the enhanced rate. These two facts are quite unexpected, considering

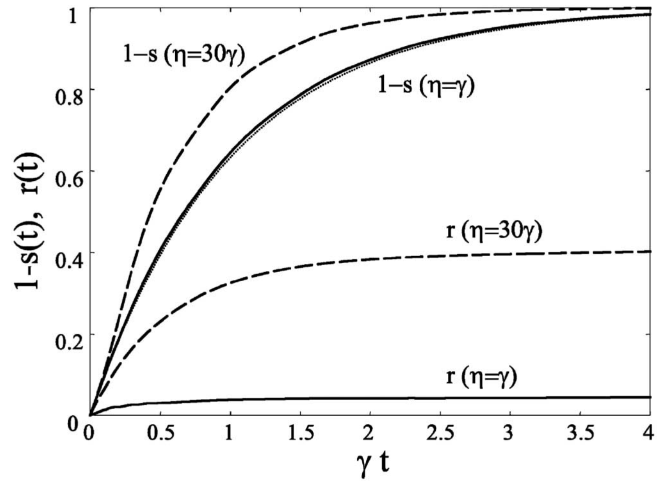

FIG. 2. Temporal evolutions of $1 - s(t)$ and $r(t)$. The solid and broken lines show the results for $\eta = \gamma$ (when the response of the detector is slow) and $\eta = 30\gamma$ (when the response of the detector is fast). The inactive bandwidth Δ is 10γ. The thin dotted line shows the decay probability for the unobserved case, where $1 - s(t) = 1 - e^{-\gamma t}$.

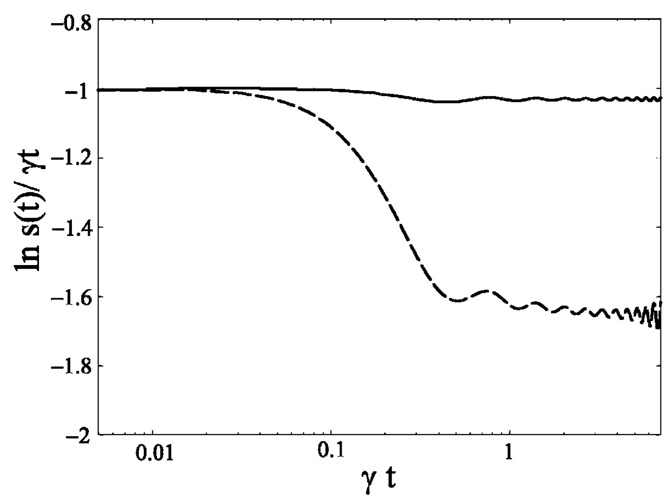

FIG. 3. Temporal evolution of $\ln s(t)/\gamma t$, where $\Delta = 10\gamma$, and $\eta = \gamma$ (solid line) and 30γ (broken line). The system decays with the unobserved decay rate γ for $t \lesssim \Delta^{-1}$, and with the enhanced decay rate $2\pi|g_\Omega|^2$ for $t \gtrsim \Delta^{-1}$.

that the energy of the emitted photon lies almost completely in the inactive band of the detector and therefore that the detector seemingly cannot touch the target system.

In order to understand these curious results, we transform the Hamiltonian $\mathcal{H} = \mathcal{H}_s + \mathcal{H}_{sd}$ into the following form:

$$\mathcal{H} = \Omega|x\rangle\langle x| + \int d\mu[(g_\mu|x\rangle\langle g|B_\mu + \text{H.c.}) + \mu B_\mu^\dagger B_\mu] + \mathcal{H}', \tag{5}$$

where B_μ is composed by linear combination of $b_{\boldsymbol{k}}$ and $c_{\boldsymbol{k}\omega}$ and is normalized by $[B_\mu, B_{\mu'}^\dagger] = \delta(\mu - \mu')$, and $\mathcal{H}'$ contains the terms which do not interact with the atom [14]. In Eq. (5), the atom is coupled to a one-dimensional continuum of B_μ, and the decay dynamics is completely determined by the coupling function g_μ, which is called a form factor. $|g_\mu|^2$ is given by

$$|g_\mu|^2 = \int d\boldsymbol{k}|g_{\mu,\boldsymbol{k}}|^2, \tag{6}$$

$$|g_{\mu,\boldsymbol{k}}|^2 = |\xi_{\boldsymbol{k}}|^2 \frac{\eta_{\boldsymbol{k}}/2\pi}{|\mu - k - i\eta_{\boldsymbol{k}}/2|^2}. \tag{7}$$

Renormalization of the form factor by measurement, according to Eqs. (6) and (7), is visualized in Fig. 4. $|g_{\mu,\boldsymbol{k}}|^2$ represents the contribution of the photon $\boldsymbol{k}$ to the form factor $|g_\mu|^2$. When photons are not measured, i.e., $\eta_{\boldsymbol{k}} \to 0$, $|g_{\mu,\boldsymbol{k}}|^2$ reduces to a delta function, $|g_{\mu,\boldsymbol{k}}|^2 = |\xi_{\boldsymbol{k}}|^2\delta(\mu - k)$. Contrarily, when photons are measured, lifetimes of photons become finite as an inevitable result of a measurement, and $|g_{\mu,\boldsymbol{k}}|^2$ is broadened to be a Lorentzian with width $\eta_{\boldsymbol{k}}$, satisfying a sum rule, $\int d\mu|g_{\mu,\boldsymbol{k}}|^2 = |\xi_{\boldsymbol{k}}|^2$. The form factor is composed by accumulating $|g_{\mu\boldsymbol{k}}|^2$ by Eq. (6). It should be noted that the above renormalization is mathematically rigorous and holds for general functional forms of $\xi_{\boldsymbol{k}}$ and $\eta_{\boldsymbol{k}}$. The usual Zeno and anti-Zeno effects (with finite jump time) are understandable through this renormalization mechanism [14].

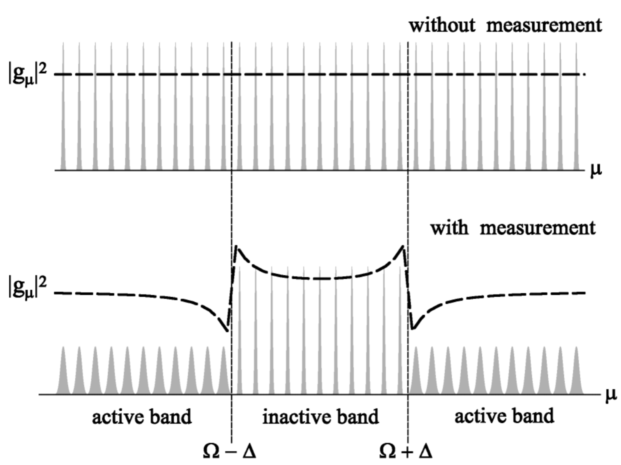

FIG. 4. Renormalization of the form factor $|g_\mu|^2$ by the measurement (schematic). Without the measurement, $|g_{\mu,\boldsymbol{k}}|^2$ is a delta function (sharp spikes in the upper figure). If the photon energy k lies in the active band, $|g_{\mu,\boldsymbol{k}}|^2$ is broadened by the measurement (duller spikes in the lower figure). The form factor $|g_\mu|^2$ is composed by accumulating $|g_{\mu,\boldsymbol{k}}|^2$.

In our model, the form factor reduces to a constant function $|g_\mu|^2 = \gamma/2\pi$ when the system is not measured. By the false measurement, $|g_{\mu,\boldsymbol{k}}|^2$ is broadened if the photon energy k lies in the active band, while it is kept unchanged (remains a delta function) if k lies in the inactive band, as shown in Fig. 4. As a result, $|g_\mu|^2$ is renormalized into the following form: $|g_\mu|^2 = (\gamma/2\pi^2)\{\pi + \pi\theta(\Delta - |\mu - \Omega|) + \arctan[2(\mu - \Omega - \Delta)/\eta] - \arctan[2(\mu - \Omega + \Delta)/\eta]\}$, where $\theta(x)$ is a step function. $|g_\mu|^2$ is plotted in Fig. 5 for three values of η/Δ. It is observed that $|g_\mu|^2$ is increased in the inactive band, which is compensated by a decrease in the active band. We can confirm that false measurements necessarily result in an increase of the form factor $|g_\mu|^2$ inside the inactive band, which implies that false measurements always result in enhancement of decay (anti-Zeno effect). The form of $|g_\mu|^2$ is determined by the ratio η/Δ. When $\eta/\Delta \ll 1$, modification of $|g_\mu|^2$ occurs only around the edge of the inactive band, $|\mu - (\Omega \pm \Delta)| \lesssim \eta$. Contrarily, when $\eta/\Delta \gtrsim 1$, $|g_\mu|^2$ is increased globally in the inactive band.

The form factor $|g_\mu|^2$ explains the two-step behavior observed in Fig. 3. Using the lowest-order perturbation to Eq. (5), the decay probability is given by $1 - s(t) = t^2 \int d\mu|g_\mu|^2 \text{sinc}^2[(\mu - \Omega)t/2]$. Remembering that the sinc function is large only in $|\mu - \Omega| \lesssim 2\pi t^{-1}$, the right-hand side is approximated by $2\pi|g_\infty|^2 t (= \gamma t)$ for $t \ll \Delta^{-1}$, and by $2\pi|g_\Omega|^2 t$ for $t \gg \Delta^{-1}$. Thus, the decay rate is enhanced at $t \sim \Delta^{-1}$ from the natural rate γ to the enhanced rate,

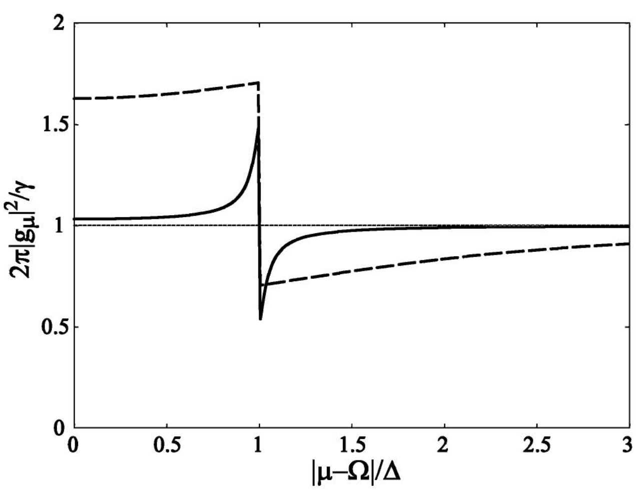

FIG. 5. Plot of the form factor $|g_\mu|^2$ under the measurement, for $\eta/\Delta = 0$ (thin dotted line), 0.1 (solid line), and 3 (broken line).

$$\bar{\gamma} = 2\pi|g_\Omega|^2 = \gamma[2 - 2\pi^{-1}\arctan(2\Delta/\eta)]. \quad (8)$$

The increase of the photodetection probability $r(t)$, observed in Fig. 2, is also explained in terms of $|g_\mu|^2$. The increase of $|g_\mu|^2$ inside the detection band is ascribed to a new decay channel opened by measurement; the atom can decay to an excitation in the detector via off-resonant photons lying in the active band. The atomic decay can be detected if this decay channel is used. Besides this new channel, there remains the original decay channel, where the atom decays to a photon within the natural linewidth. The detector cannot catch the emitted photon in this case. Thus, the probability to use the new channel may be regarded as the final photodetection probability $r(\infty)$, which is estimated by

$$r(\infty) \simeq \frac{\bar{\gamma} - \gamma}{\bar{\gamma}} = \frac{1 - 2\pi^{-1}\arctan(2\Delta/\eta)}{2 - 2\pi^{-1}\arctan(2\Delta/\eta)}. \quad (9)$$

Equations (8) and (9) agree well with the numerical results.

Now it is clarified that enhancement of decay and increase of the detection probability are indivisible results of the new decay channel opened by measurement. The condition for inducing considerable effects is that $\bar{\gamma}$ is significantly larger than the original value γ, which is accomplished when $\eta/\Delta \gtrsim 1$, as indicated by Eq. (8) [19]. We can understand this condition intuitively as follows: The lifetime of a virtually emitted photon in the detection band is estimated by $\delta t \sim (\delta E)^{-1} \sim \Delta^{-1}$, using the uncertainty principle. The anti-Zeno effect takes place if the detector response $\tau(\sim\eta^{-1})$ is quick enough to fix a virtual photon, which is accomplished by $\tau \lesssim \delta t$; i.e., $\eta/\Delta \gtrsim 1$.

In the limit of infinitely quick response, $\eta \to \infty$, the decay rate is doubled and the detection probability reaches a half in our case. These values are, of course, strongly dependent on the original form factor without the measurement. For example, when ξ_k is zero around the atomic transition energy but is nonzero in other regions, the effects of measurement appear more drastically. In this case, the atom does not decay $[s(\infty) \simeq 1]$ when photons are not observed. However, the decay becomes possible by detecting virtually emitted photons, and the decay can be detected almost perfectly. Thus, false measurements may affect the dynamics of quantum systems even qualitatively.

In summary, we have investigated the effects of a *false* measurement on radiative decay, where the active energy band of a photodetector does not match the energy of emitted photons (Fig. 1). When the detector response is slow, the detector can hardly catch the photon and the decay rate is almost unchanged from the unobserved case (solid curves in Figs. 2 and 3), as expected. However, when the response is quick, the detectability is largely increased and the decay is accelerated (broken curves in Figs. 2 and 3). These two facts are explained in terms of renormalization of the form factor, which inevitably occurs as a counteraction of photodetection. The renormalization is generally expressed by Eqs. (6) and (7). In the case of false measurements, $|g_\Omega|^2$ (form factor at transition energy) is necessarily increased, implying that the decay is always accelerated. $|g_\Omega|^2$ is related to the decay rate and the detectability by Eqs. (8) and (9), respectively.

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