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Conditional sign flip of two photons without pulse distortion

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We theoretically demonstrate a deterministic conditional sign flip of a two-photon pulse that preserves the pulse profile. The nonlinear optical system used is a V-type three-level atom confined in a cavity, and the input photons are down-converted twin photons having temporal entanglement. The input and the output pulse profiles are nearly perfectly preserved when the reciprocal bandwidth of the input photons is tuned to the atomic linewidth.

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I. INTRODUCTION

Photons interact very weakly with environmental degrees of freedom and thus can maintain quantum coherence for a long time. Consequently, photons have been widely used for fundamental tests of quantum mechanics [1–3], and they are regarded as one of the most promising candidates for implementing unique quantum-mechanical technologies [4–7]. However, one problem in using photons in device applications is that it is difficult to control one photon by using another photon since the nonlinear interaction between single photons is negligibly small [8]. Several strategies have been proposed for overcoming the weakness of the nonlinear interaction between single photons; such strategies include the probabilistic realization of conditional gates by linear optical elements and measurements [9] and amplification of nonlinear effects by a strong classical field [10].

Extensive efforts have also been made to realize a giant optical nonlinearity that is sensitive to individual photons [11–13]. In particular, the use of a few-level quantum system seems highly promising [14–17], and considerable nonlinear effects have actually been obtained by using extremely weak input fields at the single-photon level [18]. In order to achieve strong interaction between single photons, the input photons must be in a highly dispersive frequency region close to the resonance of the system. However, this generally greatly distorts the photonic pulse profile in the propagation direction. Such distortion in the pulse profile reduces the fidelity of the photons and should thus be avoided when constructing single-photon devices. Previous multimode quantum-optical analyses [19–22] have demonstrated that few-level quantum systems can generate substantial nonlinear interactions between single photons. However, the control of the output pulse profile is essential for actual device applications.

In this study, we theoretically show that the deterministic conditional sign flip of a two-photon pulse can be performed without distorting the pulse profile. The elements required are a V-type three-level atom confined in a cavity and a twin photon pair having temporal entanglement, both of which are available in current quantum-optics experiments. These results imply that a *deterministic* optical controlled-Z gate [23] can be constructed from existent optical elements.

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II. SYSTEM

A. Optical circuit

The optical circuit considered in this study is illustrated in Fig. 1. Two photons are input into the circuit from the optical paths P_1 and P_2. The polarization beam splitters are arranged to transmit horizontally (H) polarized photons and to reflect vertically (V) polarized photons. The paths P_3, P_4, and P_5 incorporate a nonlinear cavity, which contains a V-type three-level quantum system as the nonlinear material, as depicted in Fig. 2. These cavities do not affect the polarization of photons. The polarizations of the output photons (P_6 and P_7) are thus identical to those of the input photons (P_1 and P_2). When the input polarization state is $|VH\rangle_{in}$ (i.e., V polarization in P_1 and H polarization in P_2), two input photons are forwarded to the same cavity. In contrast, when the input polarization state is $|HH\rangle_{in}$, $|VV\rangle_{in}$, or $|HV\rangle_{in}$, two input photons are forwarded to different cavities. Therefore, we should investigate the one- and the two-photon dynamics occurring in the nonlinear cavity.

B. Nonlinear cavity

The structure of a nonlinear cavity is illustrated in Fig. 2. The $|0\rangle \leftrightarrow |1\rangle$ ($|0\rangle \leftrightarrow |2\rangle$) transition in the atom is assisted by a H- (V-) polarized cavity photon. The Hamiltonian $\mathcal{H}$ of the

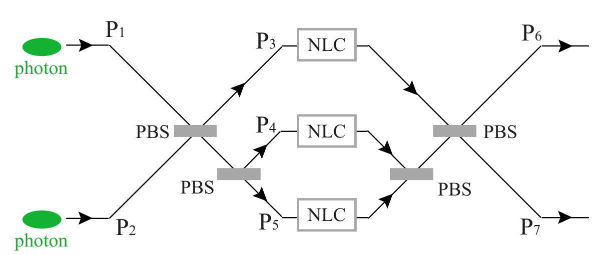

FIG. 1. (Color online) The optical circuit considered in this study. PBS: polarization beam splitter; NLC: nonlinear cavity. The polarizations of the two input photons (P_1 and P_2) are conserved at the output ports (P_6 and P_7).

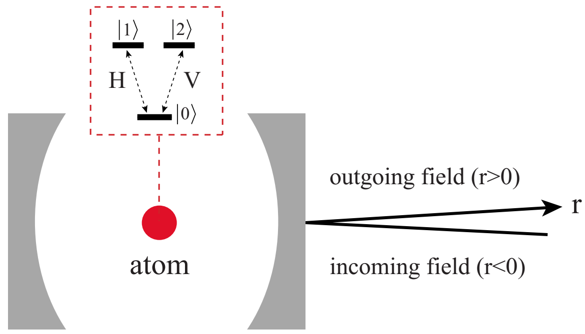

FIG. 2. (Color online) Structure of the nonlinear cavity. A V-type three-level atom is placed inside a one-sided cavity. The $|0\rangle\leftrightarrow|1\rangle$ ($|0\rangle\leftrightarrow|2\rangle$) transition in the atom is assisted by a H- (V-) polarized photon.

nonlinear cavity including the external field is given by (putting $\hbar=c=1$)

$$\mathcal{H}=\mathcal{H}_a+\mathcal{H}_{ac}+\mathcal{H}_c+\mathcal{H}_{ce}+\mathcal{H}_e, \tag{1}$$

$$\mathcal{H}_a=\Omega(\sigma_{11}+\sigma_{22}), \tag{2}$$

$$\mathcal{H}_{ac}=g(\sigma_{10}b_H+\sigma_{20}b_V+\text{H.c.}), \tag{3}$$

$$\mathcal{H}_c=\Omega(b_H^\dagger b_H+b_V^\dagger b_V), \tag{4}$$

$$\mathcal{H}_{ce}=\int dk\sqrt{\frac{\kappa}{2\pi}}(b_H^\dagger b_{H,k}+b_V^\dagger b_{V,k}+\text{H.c.}), \tag{5}$$

$$\mathcal{H}_e=\int dk(kb_{H,k}^\dagger b_{H,k}+kb_{V,k}^\dagger b_{V,k}). \tag{6}$$

The parameters have the following meanings. Ω is the cavity resonance frequency, which is identical to the atomic transition frequency, g is the atom-cavity coupling, and κ is the escape rate of a cavity photon into the external field. Note that the bad-cavity regime of $\kappa \gg g$ is assumed throughout this study. The operators have the following meanings. σ_{ij} $(=|i\rangle\langle j|)$ is the atomic transition operator, b_H (b_V) is the annihilation operator for the H- (V-) polarized cavity photon, $b_{H,k}$ ($b_{V,k}$) is the annihilation operator for the H- (V-) polarized external field with wave number k. As Fig. 2 shows, we define the spatial coordinate r along the propagation direction of the incoming and the outgoing photons and assign the negative (positive) region to the incoming (outgoing) field. The real-space representation $\widetilde{b}_{H,r}$ of the external-field operator is the Fourier transform of $b_{H,k}$: $\widetilde{b}_{H,r}=(2\pi)^{-1/2}\int dk\, e^{ikr}b_{H,k}$.

III. TEMPORAL EVOLUTION OF PHOTONS

Using this Hamiltonian, we can relate the pulse profiles of incoming and outgoing photons. The following three situations are relevant in the current situation: (i) only a H-polarized photon enters the cavity, (ii) only a V-polarized photon enters the cavity, and (iii) H-polarized and V-polarized photons simultaneously enter the cavity. The one-photon propagators that connect the input and the output photons in cases (i) and (ii) are defined by

$$G^H(r,r')=\langle g|\widetilde{b}_{H,r}(t)\widetilde{b}_{H,r'}^\dagger(0)|g\rangle, \tag{7}$$

$$G^V(r,r')=\langle g|\widetilde{b}_{V,r}(t)\widetilde{b}_{V,r'}^\dagger(0)|g\rangle, \tag{8}$$

where $A(t)=e^{i\mathcal{H}t}A(0)e^{-i\mathcal{H}t}$ (Heisenberg picture); $\mathcal{H}$ is the overall Hamiltonian of Eq. (1); $|g\rangle$ is its ground state; t is the final moment; and r' and r, respectively, denote the space variables of the incoming and the outgoing photons, satisfying $r'<0<r<t$. The t dependences of G^H and G^V are not shown explicitly. The two-photon propagator G^{VH}, which corresponds to case (iii), is defined similarly by

$$G^{VH}(r_1,r_2,r_1',r_2')=\langle g|\widetilde{b}_{V,r_1}(t)\widetilde{b}_{H,r_2}(t)\widetilde{b}_{V,r_1'}^\dagger(0)\widetilde{b}_{H,r_2'}^\dagger(0)|g\rangle, \tag{9}$$

where r_1 and r_1' (r_2 and r_2') are the space variables for the V- (H-) polarized photon. These propagators can be calculated by solving the Heisenberg equation. In the bad-cavity regime of $\kappa \gg g$, they are given by

$$\begin{aligned}G^H(r,r')&=G^V(r,r')\\&=-\delta(r-r'-t)+\Gamma e^{(i\Omega+\Gamma/2)(r-r'-t)}\theta(r'+t-r),\end{aligned} \tag{10}$$

$$\begin{aligned}G^{VH}(r_1,r_2,r_1',r_2')&=G^V(r_1,r_1')G^H(r_2,r_2')\\&\quad-\Gamma^2\prod_{j=1,2}e^{(i\Omega+\Gamma/2)(r_j-r_j'-t)}\theta(r_j'+t-R),\end{aligned} \tag{11}$$

where $\Gamma(=4g^2/\kappa)$ is the atomic natural linewidth and $R=\max(r_1,r_2)$ in Eq. (11).

We now return to the optical circuit of Fig. 1. The wave function of two input photons (P_1 and P_2) is assumed to be independent of their polarizations and is denoted by $f_{in}(r_1',r_2')$, where r_1' and r_2', respectively, denote the spatial coordinates of the P_1 and the P_2 photons. When the input polarization state is $|HH\rangle_{in}$, $|VV\rangle_{in}$, or $|HV\rangle_{in}$, two input photons are forwarded to different cavities and thus evolve independently. The wave functions of the output photons (P_6 and P_7) are then, respectively, given by

$$f_{out}^{HH}(r_1,r_2)=\int dr_1'dr_2'\, G^H(r_1,r_1')G^H(r_2,r_2')f_{in}(r_1',r_2'), \tag{12}$$

$$f_{out}^{VV}(r_1,r_2)=\int dr_1'dr_2'\, G^V(r_1,r_1')G^V(r_2,r_2')f_{in}(r_1',r_2'), \tag{13}$$

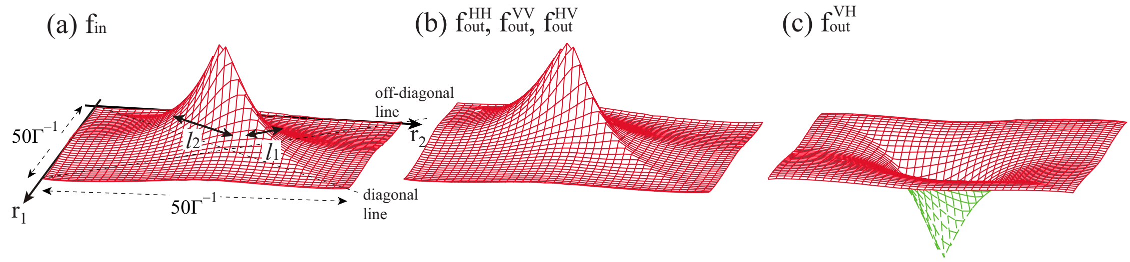

FIG. 3. (Color online) Three-dimensional plots of (a) f_{in}, (b) $f_{out}^{HH}(=f_{out}^{VV}=f_{out}^{HV})$, and (c) f_{out}^{VH}. $l_1=\Gamma^{-1}$ and $l_2=5\Gamma^{-1}$. The natural phase factor of $e^{i\Omega(r_1+r_2)}$ is neglected. The sign of the output wave function is flipped only when the input polarization is $|VH\rangle_{in}$.

$$f_{out}^{HV}(r_1,r_2)=\int dr_1'dr_2'\, G^H(r_1,r_1')G^V(r_2,r_2')f_{in}(r_1',r_2'). \tag{14}$$

Since $G^H=G^V$ [see Eq. (10)], $f_{out}^{HH}=f_{out}^{VV}=f_{out}^{HV}$. In contrast, when the input polarization state is $|VH\rangle_{in}$, two input photons are forwarded to the same cavity. The wave function of the output photons is then given by

$$f_{out}^{VH}(r_1,r_2)=\int dr_1'dr_2'\, G^{VH}(r_1,r_2,r_1',r_2')f_{in}(r_1',r_2'). \tag{15}$$

IV. QUANTUM STATES OF OUTPUT PHOTONS

A. Pulse profile of input photons

In the following part of this study, by assuming a specific form of f_{in}, we evaluate the output wave functions f_{out}^{HH} $(=f_{out}^{VV}=f_{out}^{HV})$ and f_{out}^{VH}. We employ the following form of f_{in}:

$$f_{in}(r_1,r_2)=\mathcal{N}\exp\left(-\frac{|r_1-r_2|}{2l_1}-\frac{|r_1+r_2|}{2l_2}\right), \tag{16}$$

where $\mathcal{N}=(2l_1l_2)^{-1/2}$ is a normalization constant to satisfy $\int dr_1dr_2|f_{in}|^2=1$ and $l_2(l_1)$ denotes the pulse length in the diagonal (off-diagonal) direction [see Fig. 3(a)]. Note that the input photons are assumed to be in resonance with the atom, and the natural phase factor of $e^{i\Omega(r_1+r_2)}$ is dropped here and hereafter for notational simplicity. Since $\langle|r_1-r_2|\rangle=l_1$, l_1 represents the correlation length between two photons. The correlation parameter ρ, defined by $\rho=\langle r_1r_2\rangle/\sqrt{\langle r_1^2\rangle\langle r_2^2\rangle}$, is given by

$$\rho=\frac{l_2^2-l_1^2}{l_1^2+l_2^2}. \tag{17}$$

For twin photons generated by parametric down conversion, $l_2\gg l_1$ and therefore $\rho\sim 1$ are usually satisfied. Namely, two photons are entangled temporally.

B. Conditional sign flip

We first examine the long pulse limit of $l_2\to\infty$ while keeping l_1 finite. In this limit, nonlinear effects will disappear if two photons without temporal entanglement are input, since the mean time separation between two photons becomes infinite. In contrast, when temporally entangled photons are input, as in the present case, considerable nonlinear effects can occur even in the long pulse limit. As observed in Eq. (16), in the $l_2\to\infty$ limit, f_{in} becomes a function of only a single variable, $|r_1-r_2|$. Then, f_{out}^{HH} and f_{out}^{VH} also become functions of $|r_1-r_2|$ only. Based on this observation and Eqs. (10)–(15), the output wave functions are recast into the following forms:

$$f_{out}^{HH}(r_1,r_2)=f_{in}(r_1,r_2), \tag{18}$$

$$f_{out}^{VH}(r_1,r_2)=f_{in}(r_1,r_2)+\mathcal{C}e^{-\Gamma|r_1-r_2|/2}, \tag{19}$$

where the coefficient $\mathcal{C}$ in Eq. (19) is given by

$$\mathcal{C}=-2\Gamma\int_0^\infty d\lambda\, e^{-\Gamma\lambda/2}f_{in}(\lambda), \tag{20}$$

where $\lambda=|r_1-r_2|$. In particular, when $l_1=\Gamma^{-1}$, Eqs. (19) and (20) are reduced to the following form:

$$f_{out}^{VH}(r_1,r_2)=-f_{in}(r_1,r_2). \tag{21}$$

Equation (18) implies that the output state is unchanged from the input one (except for the translational motion), namely, $|HH\rangle_{out}=|HH\rangle_{in}$, $|VV\rangle_{out}=|VV\rangle_{in}$, and $|HV\rangle_{out}=|HV\rangle_{in}$. In contrast, Eq. (21) implies the nonlinear sign flip, $|VH\rangle_{out}=-|VH\rangle_{in}$. Thus, for a suitable choice of the input pulse profile ($l_1=\Gamma^{-1}$ and $l_2\to\infty$), the conditional sign flip of a two-photon pulse can be performed without distorting the pulse profile between the input and the output.

C. Numerical results

Since the pulse lengths are finite in reality, we examine here the effects of the finiteness of l_2 numerically. Three-dimensional plots of f_{in}, f_{out}^{HH}, and f_{out}^{VH} are presented in Fig. 3, for $l_1=\Gamma^{-1}$ and $l_2=5\Gamma^{-1}$. It is observed that f_{out}^{HH} is almost unchanged from f_{in}, whereas sign flip occurs in f_{out}^{VH}. The sectional plots of f_{in}, f_{out}^{HH}, and $-f_{out}^{VH}$ on the diagonal and the off-diagonal lines [see Fig. 3(a)] are presented in Fig. 4. As expected from analytical results, $f_{in}\simeq f_{out}^{HH}\simeq -f_{out}^{VH}$, and these three functions become closer as l_2 is increased. In contrast, the results for the $l_1\neq\Gamma^{-1}$ case are presented in Fig. 5. Although f_{out}^{HH} is nearly unchanged from f_{in}, $-f_{out}^{VH}$ differs from f_{in} even when l_2 is large.

In order to quantify the distortion of the pulse profile, we observe the overlap α between $|HH\rangle_{out}$ and $|VH\rangle_{out}$, defined by

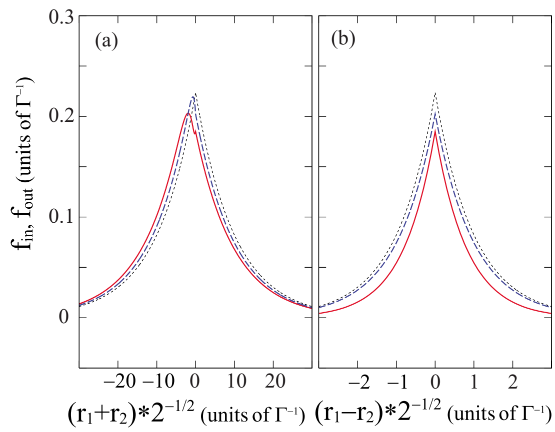

FIG. 4. (Color online) Sectional plots of f_{in} (thin dotted line), f_{out}^{HH} (solid line), and $-f_{out}^{VH}$ (dashed line), on the (a) diagonal and (b) off-diagonal lines [see Fig. 3(a)]. $l_1=\Gamma^{-1}$ and $l_2=10\Gamma^{-1}$. The overlap between f_{out}^{HH} and f_{out}^{VH} is $\alpha=-0.98$.

$$\alpha=\int dr_1 dr_2 [f_{out}^{HH}(r_1,r_2)]^* f_{out}^{VH}(r_1,r_2). \tag{22}$$

By definition, α is a complex value satisfying $|\alpha|\leq 1$ and is related to the fidelity $\mathcal{F}$ between $|HH\rangle_{out}$ and $|VH\rangle_{out}$ by $\mathcal{F}=|\alpha|^2$. A complete sign flip without pulse distortion corresponds to $\alpha=-1$. In Fig. 6, the dependence of α on l_2 is plotted by fixing l_1. Note that α necessarily takes a real value in the present case of resonant input. It is observed that a high-fidelity sign flip is attained when $l_1=\Gamma^{-1}$ and l_2 is large. For example, $\mathcal{F}\geq 0.99$ ($\alpha\leq -0.995$) is attained when $l_2\geq 21.4\Gamma^{-1}$.

V. CONTROLLED TWO-PHOTON GATE

It was clarified in Sec. IV that, under the conditions of $l_1=\Gamma^{-1}$ and $l_2\gg l_1$, the conditional sign flip of two-photon pulse can be achieved without pulse distortion. Namely, $|HH\rangle_{in}\to|HH\rangle_{out}$, $|HV\rangle_{in}\to|HV\rangle_{out}$, $|VV\rangle_{in}\to|VV\rangle_{out}$, and $|VH\rangle_{in}\to-|VH\rangle_{out}$. By assigning the computational basis

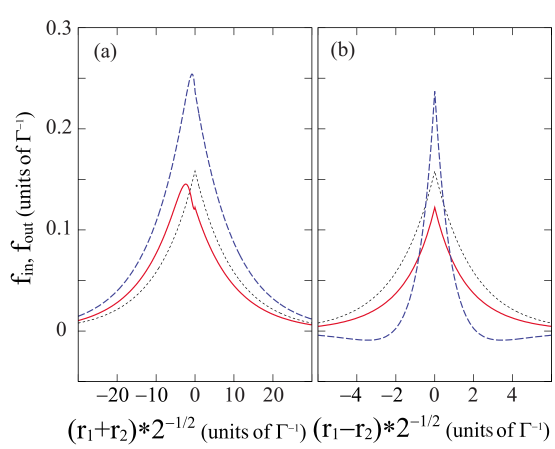

FIG. 5. (Color online) The same plots as those in Fig. 4, for $l_1=2\Gamma^{-1}$ and $l_2=10\Gamma^{-1}$. The overlap between f_{out}^{HH} and f_{out}^{VH} is $\alpha=-0.76$.

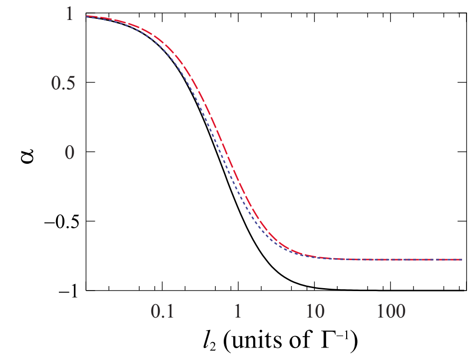

FIG. 6. (Color online) Plot of α as a function of l_2. $l_1=\Gamma^{-1}$ (solid line), $2\Gamma^{-1}$ (dashed line), and $0.5\Gamma^{-1}$ (dotted line). High-fidelity operation ($\alpha\leq-0.995$, for example) is attained when $l_1=\Gamma^{-1}$ and $l_2>21.4\Gamma^{-1}$.

states to the polarization states of photons, this dynamics realizes a controlled-Z gate. This gate can be readily switched to a controlled-NOT gate with the help of two Hadamard gates [23], which is realized by quarter-wave plates.

In former studies, it was shown that the temporal entanglement in down-converted photons can be turned into a controlled-NOT gate by only *linear* optics [24]. The difference between such linear schemes and the current one lies in the success probability of the gate. In linear schemes, the gate operation succeeds probabilistically in general [9]. In contrast, in nonlinear schemes including the current one, the gate operation succeeds with certainty [12]. The merit of the current scheme lies in the deterministic and high-fidelity operation of the gate.

VI. REMARKS

Three final remarks are in order. (i) In the Hamiltonian (1) of the nonlinear cavity, atomic radiative damping into noncavity modes (usually denoted by γ) is assumed to be negligible. Such a system, called a *one-dimensional* atom, has actually been realized by existent Fabry-Pérot or microtoroid resonators [18,25]. Further suppression of γ would be possible by using photonic band-gap materials. (ii) Since Γ^{-1} is on the order of nanoseconds in typical atomic cavity-QED systems [25], the correlation time l_1 of twin photons should be tuned to this order. Such narrow-band photon pairs can be generated by a cavity-enhanced down conversion [26]. (iii) The results for the uncorrelated input photons correspond to the cases of $l_1=l_2$, because the temporal correlation ρ vanishes in these cases [see Eq. (17)]. When $l_1=l_2=\Gamma^{-1}$, for example, we can observe in Fig. 6 that $\alpha=-0.4$ (fidelity $\mathcal{F}=|\alpha|^2=0.16$), indicating a significant pulse distortion. Thus, there is a trade-off relation between the input correlation and the output fidelity.

VII. SUMMARY

In summary, it is theoretically demonstrated that conditional sign flip of a two-photon pulse can be performed deterministically while preserving the pulse profile of the input photons. The elements required are a V-type three-level atom confined in a cavity and a temporally entangled photon pair. These results demonstrate the potential of using existent quantum-optical elements to construct quantum-mechanical devices that are based on single-photon optical nonlinearity.

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