Quantum Zeno and anti-Zeno effects by indirect measurement with finite errors

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We study the quantum Zeno effect and the anti-Zeno effect in the case of ''indirect'' measurements, where a measuring apparatus does not act directly on an unstable system, for a realistic model with finite errors in the measurement. A general and simple formula for the decay rate of the unstable system under measurement is derived. In the case of a Lorentzian form factor, we calculate the full time evolutions of the decay rate, the response of the measuring apparatus, and the probability of errors in the measurement. It is shown that not only the response time but also the detection efficiency plays a crucial role. We present the prescription for observing the quantum Zeno and anti-Zeno effects, as well as the prescriptions for avoiding or calibrating these effects in general experiments.

It was predicted in a classic paper [1] that repeated measurements on a quantum unstable system, at time intervals τ_m, suppress the decay of the system for small τ_m—the so-called quantum Zeno effect (QZE). It was assumed there that each measurement is completely ideal, i.e., it takes only an infinitesimal time, there is no error in the measurement, and the postmeasurement state is exactly given by the projection postulate. However, any physical experiments do not satisfy all of these assumptions; hence more careful studies have been desired.

According to the general measurement theory, not only the unstable system in question but also a part of the measuring apparatus should be treated as a quantum system subject to the Schrödinger equation [2–6]. It was clarified by such theories that the response time τ_r of the apparatus corresponds to τ_m of Ref. [1]. However, effects of the errors in the measurement are yet to be explored, because the probability ε of getting an erroneous result is determined not only by a finite response time τ_r but also by the detection efficiency $1 - \varepsilon_\infty$ (i.e., the apparatus occasionally fails to detect the decay even after an infinitely long waiting time). Moreover, these pioneering theories, as well as pioneering experiments [7–9], studied the case of ''direct'' measurements, where the apparatus acts directly on the unstable system (e.g., shines laser light to excited atoms). In such a case, however, the dynamics of the unstable system would be affected by the apparatus even if the unstable system were a classical system, and thus the Zeno effect in direct measurements might not be peculiar to quantum systems. Hence, the most interesting case of ''indirect'' measurements is yet to be explored, where the apparatus does not act directly on the unstable system, but detects a signal mediated by some field. A promising theory of an indirect measurement was developed by Schulman [10]. However, since his model was an abstract one, application to real physical systems is not straightforward. For example, it did not clarify the conditions to observe *acceleration* of the decay by measurements, the anti-Zeno effect (AZE), which has also been attracting much attention [9,11,12]. To apply real experiments on QZE and AZE, more realistic models should be analyzed. Such analysis should also be important to *general* experiments, because the QZE or the AZE might slip in results of experiments not designed to detect it. The purpose of this paper is to present a theory that satisfies all these requirements, using a realistic model of an indirect measurement with finite errors.

The total system in our model is composed of three parts: (i) an unstable two-level system, which is initially in the excited state $|x\rangle$ with the transition energy Ω_0 to the ground state $|g\rangle$, (ii) a field whose eigenmodes are labeled by a wave vector $\boldsymbol{k}$ with any dimension, a quantum of which is emitted by the unstable system when it decays to $|g\rangle$, and (iii) a measuring apparatus that detects the emitted quantum by absorbing it, from that an observer gets to know the decay of the unstable system. Similar models of measurement have been frequently used in quantum optics, where the unstable system is an excited atom that emits a photon upon decay, and the photon is detected by a photodetector such as a photomultiplier. Therefore, we hereafter call (i), (ii), and (iii) as an atom, photon, and detector, respectively, although the theory is applicable to other systems as well. In this model, neither a projection operator nor the interaction Hamiltonian of the detector acts on the atom. Moreover, the measurement is of negative-result type, where no signal is detected until the atom decays: The QZE or the AZE occurs just by waiting for the decay.

The role of a detector is to convert a photon into other kinds of elementary excitations, which finally yield macroscopic signals after magnification processes, usually obeying classical mechanics. As a model of the relevant part of the detector, we assume elementary excitations (e.g., electron-hole pairs) with a continuous spectrum, into which photons are converted. By taking the energy of $|g\rangle$ zero, the Hamiltonian of the system is taken as follows (with $\hbar = 1$);

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$$\mathcal{H} = \Omega_0 |x\rangle\langle x| + \mathcal{H}_1 + \mathcal{H}_2, \tag{1}$$

$$\mathcal{H}_1 = \int d\mathbf{k} [(g_{\mathbf{k}} |x\rangle\langle g| b_{\mathbf{k}} + \text{H.c.}) + \epsilon_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}}], \tag{2}$$

$$\mathcal{H}_2 = \int\int d\mathbf{k} d\omega [(\zeta_{\mathbf{k}\omega} b_{\mathbf{k}}^\dagger c_{\mathbf{k}\omega} + \text{H.c.}) + \omega c_{\mathbf{k}\omega}^\dagger c_{\mathbf{k}\omega}]. \tag{3}$$

Here, $b_{\mathbf{k}}$ is the annihilation operator for a photon with energy $\epsilon_{\mathbf{k}}$ and $g_{\mathbf{k}}$ represents the atom-photon coupling. The photonic dispersion relation can be nonlinear ($\epsilon_{\mathbf{k}} \neq |\mathbf{k}|$) as in, say, photonic crystals. Every photon mode is linearly coupled to a continuum of bosonic [13] elementary excitations, denoted by $c_{\mathbf{k}\omega}$, in the detector, with the coupling constant $\zeta_{\mathbf{k}\omega}$. The commutation relations are orthonormalized as $[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k}-\mathbf{k}')$ and $[c_{\mathbf{k}\omega}, c_{\mathbf{k}'\omega'}^\dagger] = \delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')$. The lifetime $\tau_{\mathbf{k}}$ of a photon is determined by $\zeta_{\mathbf{k}\omega}$. We will show later that $\tau_{\mathbf{k}} \simeq \tau_r$, the response time of the detector. In most experiments, the detector does not cover the whole solid angle around the atom. When a photon is emitted in the uncovered direction, it cannot be detected and has a long lifetime. As we will demonstrate later, we can encompass such realistic situations by allowing $\mathbf{k}$-dependence of $\tau_{\mathbf{k}}$. We here put $\zeta_{\mathbf{k}\omega} = \sqrt{\eta_{\mathbf{k}}}$ [14], which results in $\tau_{\mathbf{k}} = (2\pi\eta_{\mathbf{k}})^{-1}$. Equations (1)–(3) already suggest that modification of the decay rate Γ may occur through the effect of "cavity quantum electrodynamics" [15]. Our task is to study Γ as a function of relevant quantities in the measurement, such as the response time and the measurement error.

It is essential to include strong effects of the detector. We, therefore, diagonalize the photon-detector part, $\mathcal{H}_1 + \mathcal{H}_2$, using the coupled-mode operator [16], which, in this case, is given by

$$B_{\mathbf{k}\mu} = \alpha_{\mathbf{k}}(\mu) b_{\mathbf{k}} + \int d\omega \beta_{\mathbf{k}}(\mu,\omega) c_{\mathbf{k}\omega},$$

where $\alpha_{\mathbf{k}}(\mu) = \sqrt{\eta_{\mathbf{k}}}/(\mu - \epsilon_{\mathbf{k}} + i\pi\eta_{\mathbf{k}})$ and $\beta_{\mathbf{k}}(\mu,\omega) = \eta_{\mathbf{k}}/(\mu - \epsilon_{\mathbf{k}} + i\pi\eta_{\mathbf{k}})(\mu - \omega + i\delta) + \delta(\mu-\omega)$. The commutation relations for $B_{\mathbf{k}\mu}$ is given by $[B_{\mathbf{k}\mu}, B_{\mathbf{k}'\mu'}^\dagger] = \delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')$. Inversely, $b_{\mathbf{k}}$ is expressed in terms of $B_{\mathbf{k}\mu}$ as $b_{\mathbf{k}} = \int d\mu\, \alpha_{\mathbf{k}}^*(\mu) B_{\mathbf{k}\mu}$. The Hamiltonian $\mathcal{H}$ can then be rewritten as

$$\mathcal{H} = \Omega_0 |x\rangle\langle x| + \int\int d\mathbf{k} d\mu\, \mu B_{\mathbf{k}\mu}^\dagger B_{\mathbf{k}\mu}$$

$$+ \int\int d\mathbf{k} d\mu \left[\frac{\sqrt{\eta_{\mathbf{k}}} g_{\mathbf{k}}}{\mu - \epsilon_{\mathbf{k}} - i\pi\eta_{\mathbf{k}}} |x\rangle\langle g| B_{\mathbf{k}\mu} + \text{H.c.} \right].$$

We further rewrite $\mathcal{H}$, using $\bar{g}_\mu$ and $\bar{B}_\mu$ that are defined by

$$|\bar{g}_\mu|^2 = \int d\mathbf{k} \left| \frac{\sqrt{\eta_{\mathbf{k}}} g_{\mathbf{k}}}{\mu - \epsilon_{\mathbf{k}} - i\pi\eta_{\mathbf{k}}} \right|^2, \tag{4}$$

$$\bar{B}_\mu = \frac{1}{\bar{g}_\mu} \int d\mathbf{k} \frac{\sqrt{\eta_{\mathbf{k}}} g_{\mathbf{k}}}{\mu - \epsilon_{\mathbf{k}} - i\pi\eta_{\mathbf{k}}} B_{\mathbf{k}\mu}, \tag{5}$$

so that $\bar{B}_\mu$ is orthonormalized as $[\bar{B}_\mu, \bar{B}_{\mu'}^\dagger] = \delta(\mu-\mu')$. Then, the Hamiltonian finally reduces to

$$\mathcal{H} = \Omega_0 |x\rangle\langle x| + \bar{\mathcal{H}}_1 + \bar{\mathcal{H}}_2, \tag{6}$$

$$\bar{\mathcal{H}}_1 = \int d\mu [(\bar{g}_\mu |x\rangle\langle g| \bar{B}_\mu + \text{H.c.}) + \mu \bar{B}_\mu^\dagger \bar{B}_\mu], \tag{7}$$

where $\bar{\mathcal{H}}_2$ consists of coupled modes which do not interact with the atom. Now the physical meaning of $\bar{B}_\mu$ becomes apparent; it is part of the coupled modes with energy μ that interact with the atom, whereas the other part is isolated. In terms of such operators, the Hamiltonian is expressed in the renormalized form, Eqs. (6) and (7), where the atom is coupled to a single continuum of $\bar{B}_\mu$ with a coupling constant $\bar{g}_\mu$, which is called the form factor of interaction. The form factor under measurement is determined by Eq. (4), from $\epsilon_{\mathbf{k}}$, $g_{\mathbf{k}}$, and $\eta_{\mathbf{k}}$. It should be noted that there exists a sum rule $\int d\mu |\bar{g}_\mu|^2 = \int d\mathbf{k} |g_{\mathbf{k}}|^2$, which holds for any functional forms of $\eta_{\mathbf{k}}$.

We first estimate Γ under the measurement by a lowest-order perturbation in $g_{\mathbf{k}}$. To do this, it is essential to use the renormalized form $\bar{\mathcal{H}}_1$, rather than $\mathcal{H}_1$, as the interaction term. We then obtain a simple formula

$$\Gamma = 2\pi |\bar{g}_{\Omega_0}|^2 = 2\pi \int d\mathbf{k} \left| \frac{\sqrt{\eta_{\mathbf{k}}} g_{\mathbf{k}}}{\Omega_0 - \epsilon_{\mathbf{k}} - i\pi\eta_{\mathbf{k}}} \right|^2, \tag{8}$$

which should be compared with the free decay rate,

$$\Gamma_0 = 2\pi \int d\mathbf{k} |g_{\mathbf{k}}|^2 \delta(\Omega_0 - \epsilon_{\mathbf{k}}).$$

Note that Eq. (8) includes nonperturbative effects of $\eta_{\mathbf{k}}$. The formula clearly shows that the most important effect of the measurement (i.e., of finite $\eta_{\mathbf{k}}$) is to renormalize the form factor $\bar{g}_\mu$, and that the QZE (or the AZE) occurs through the renormalization. Note that the formula is general, which holds for any forms of $\epsilon_{\mathbf{k}}$, $g_{\mathbf{k}}$, and $\eta_{\mathbf{k}}$, and for any dimension of $\mathbf{k}$. It is applicable, not only to spontaneous decay of an atom, but also to many other unstable systems if their Hamiltonian can be approximated by Eqs. (1)–(3) [13]. Moreover, the formula is also applicable to the case where the detector does not cover the full solid angle, yielding the detection efficiency <1. In fact, suppose that only photons which are emitted in some solid angle S_d in the three-dimensional space are coupled to the detector, i.e., $\eta_{\mathbf{k}} = 0$ for $(\theta, \phi) \notin S_d$, where $\mathbf{k} = (k\sin\theta\cos\phi, k\sin\theta\sin\phi, k\cos\theta)$. Then, Eq. (8) yields the simple formula

$$\Gamma = 2\pi \int k^2 dk \int_{S \in S_d} dS \left| \frac{\sqrt{\eta_{\mathbf{k}}} g_{\mathbf{k}}}{\Omega_0 - \epsilon_{\mathbf{k}} - i\pi\eta_{\mathbf{k}}} \right|^2$$

$$+ 2\pi \int k^2 dk \int_{S \notin S_d} dS |g_{\mathbf{k}}|^2 \delta(\Omega_0 - \epsilon_{\mathbf{k}}), \tag{9}$$

where $dS = d\cos\theta d\phi$.

Now we embody the above general results in an example in three dimensions, in which (i) $\eta_{\mathbf{k}} = \eta \equiv (2\pi\tau)^{-1}$ for

$(\theta,\phi)\in S_d$, whereas $\eta_k=0$ for $(\theta,\phi)\notin S_d$, (ii) $\epsilon_k=k$ (we take $c=1$), and (iii) g_k takes the Lorentzian form after the angular integration;

$$k^2\int_{S\in S_d}dS|g_k|^2=(1-\varepsilon_\infty)\gamma\Delta^2/[(k-k_0)^2+\Delta^2],$$

$$k^2\int_{S\notin S_d}dS|g_k|^2=\varepsilon_\infty\gamma\Delta^2/[(k-k_0)^2+\Delta^2],$$

where $k_0\gg\Delta$. In a case where g_k depends only on k, ε_∞ is simply given by $\varepsilon_\infty=1-|S_d|/4\pi$. In general cases, however, ε_∞ depends on both S_d and g_k. It will turn out that $1-\varepsilon_\infty$ corresponds to the detection efficiency, i.e., the probability of errors approaches ε_∞ as $t\to\infty$. The meanings of γ and Δ become transparent for $\Omega_0=k_0$, i.e., when the atomic transition energy coincides with the center of the Lorentzian. For $\Delta\gg\gamma$, the decay rate of the atom is given by $2\pi\gamma$, while the transition from the initial quadratic decrease to the exponential decrease in the survival probability occurs at $t\simeq\tau_j\equiv 2/\Delta$, which is called the "jump time" [10,12]. Using Eq. (9), the renormalized lowest-order decay rate is evaluated as

$$\frac{2\pi\gamma(1-\varepsilon_\infty)\Delta\tilde{\Delta}}{(\Omega_0-k_0)^2+\tilde{\Delta}^2}+\frac{2\pi\gamma\varepsilon_\infty\Delta^2}{(\Omega_0-k_0)^2+\Delta^2}\equiv\Gamma(\eta,\varepsilon_\infty),\quad(10)$$

where $\tilde{\Delta}=\Delta+\pi\eta$. This indicates that the effect of measurement on the decay dynamics become significant only for large η satisfying $\eta\gtrsim\Delta$, i.e., $\tau_j\gtrsim(2\pi\eta)^{-1}=\tau\simeq\tau_r$, in accordance with the pervious studies [5,10].

To see what is going on, we now calculate the temporal evolution of the wave function from the initial state $|x,0,0\rangle$. This can be pursued analytically for the Lorentzian form factor. By putting $|\psi(t)\rangle=e^{-i\mathcal{H}t}|x,0,0\rangle=f(t)|x,0,0\rangle+\int dk f_k(t)|g,k,0\rangle+\int\int dk\,d\omega f_{k\omega}(t)|g,0,k\omega\rangle$, we calculate three probabilities; $s(t)=|f(t)|^2$ (survival probability of the atom), $\varepsilon(t)=\int dk|f_k(t)|^2$ (probability that the atom has decayed but the emitted photon is not absorbed by the detector), and $r(t)=\int\int dk\,d\omega|f_{k\omega}(t)|^2$ (probability that the emitted photon is absorbed). Assuming fast classical magnification processes, we can interpret $r(t)$ as the probability of getting a detector response, whereas $\varepsilon(t)$ is the probability that the detector reports an erroneous result. One of the advantages of the present theory is that all of these interesting quantities can be calculated. For example, $s(t)$ is given by $s(t)=4\pi^2|c_{123}e^{-i\omega_1 t}+c_{231}e^{-i\omega_2 t}+c_{312}e^{-i\omega_3 t}|^2$, where c_{ijk} is given by $c_{ijk}=\gamma\Delta[(\tilde{\Delta}-\epsilon_\infty\pi\eta)(\omega_i-k_0)^2+(\Delta+\epsilon_\infty\pi\eta)\Delta\tilde{\Delta}][(\omega_i-\omega_i^*)(\omega_i-\omega_j)(\omega_i-\omega_j^*)(\omega_i-\omega_k)(\omega_i-\omega_k^*)]^{-1}$, and ω_j $(j=1,2,3)$ are the solutions of the cubic equation, $(\omega_j-\Omega_0)(\omega_j-k_0+i\Delta)(\omega_j-k_0+i\tilde{\Delta})=\pi\gamma\Delta[\omega_j-k_0+i(\Delta+\epsilon_\infty\pi\eta)]$. Figure 1 plots $1-s(t),\varepsilon(t),r(t)$ for $\eta=2\gamma$. At the initial time stage $[t\ll\tau=(2\pi\eta)^{-1}]$, $\varepsilon(t)$ increases almost in parallel with $1-s(t)$, and the detection probability $r(t)[=1-s(t)-\varepsilon(t)]$ remains almost zero. Around $t\sim\tau$, the emitted photon is gradually absorbed and $r(t)$ starts to rise. Hence, the photon lifetime τ can be regarded as the
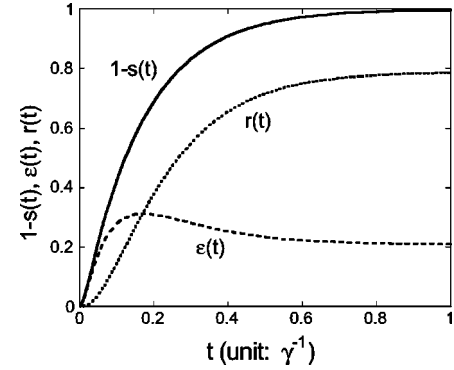

FIG. 1. The decay probability $1-s(t)$ (solid curve), the probability of getting an erroneous result $\varepsilon(t)$ (broken curve), and the probability of getting a detector response $r(t)$ (dotted curve), when $\Omega_0-k_0=0$, $\Delta=100\gamma$, $\eta=2\gamma$, and $\varepsilon_\infty=0.2$.

response time τ_r of the detector. The probabilities finally approaches the asymptotic values, $1-s(t)\to1$, $\varepsilon(t)\to\varepsilon_\infty$, and $r(t)\to1-\varepsilon_\infty$.

To see the temporal behavior of $s(t)$ more clearly, we have plotted $t^{-1}\ln s(t)$ as a function of time in Fig. 2, for several different values of the parameters. At the beginning of the decay $(t\lesssim\tau_j)$, $t^{-1}\ln s(t)$ decreases linearly as $t^{-1}\ln s(t)=-(\int dk|g_k|^2)t=-\pi\gamma\Delta t$, for any values of Ω_0-k_0 and for any values of the detector parameters η and ε_∞. Then, for $t\gtrsim\tau_j$, $t^{-1}\ln s(t)$ approaches a constant value, which is well approximated by $\Gamma(\eta,\varepsilon_\infty)$ (dotted lines). These plots demonstrate that the decay dynamics is well described, except for the initial deviation, by the exponential decay with the renormalized lowest-order decay rate. In fact, we can show analytically that formula (10) is a good approximation to the asymptotic decay rate if $\gamma\ll|\Omega_0-k_0+i\Delta|^2/\Delta$.
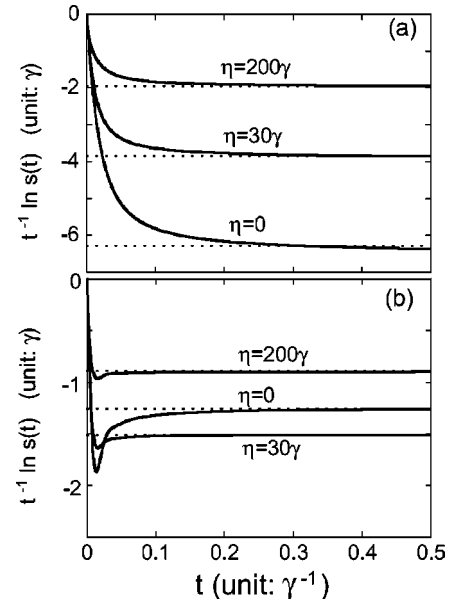

FIG. 2. Plots of $t^{-1}\ln s(t)$. $\Delta=100\gamma$ and $\varepsilon_\infty=0.2$ in both figures. Ω_0-k_0 is taken 0 ($<\Delta$) in (a), and 200γ ($>\Delta$) in (b). The dotted lines show the renormalized lowest-order decay rate, Eq. (10).
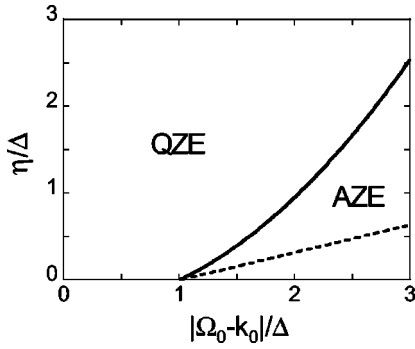
FIG. 3. The phase diagram for the QZE and the AZE for a case of Lorentzian form factor (valid for any ε_∞). The solid curve divides the QZE region and the AZE region. The dotted line shows the value of η at which the decay rate is maximized for each value of $|\Omega_0 - k_0|$.

There is a remarkable difference between Figs. 2(a) and 2(b), where different values of $|\Omega_0 - k_0|/\Delta$ are employed. In case of $|\Omega_0 - k_0|/\Delta = 0$ [see Fig. 2(a)], the decay rate decreases monotonously by increasing η, i.e., the QZE occurs at any value of η. Contrarily, in case of $|\Omega_0 - k_0|/\Delta = 2$ [Fig. 2(b)], the decay is enhanced for small η $(=30\gamma)$, while it is suppressed for large η $(=200\gamma)$. Thus, the AZE takes place for small η. By analyzing Eq. (10) as a function of $|\Omega_0 - k_0|$ and η, the phase diagram discriminating the QZE and the AZE is generated, which is shown in Fig. 3. The phase boundary (solid curve) is given by

$$\eta^{(b)} = [(\Omega_0 - k_0)^2 - \Delta^2]/\pi\Delta, \qquad (11)$$

on which the decay rate is not altered from the free rate Γ_0, while the decay rate takes the maximum value,

$$\frac{\Gamma(\eta^{(m)}, \varepsilon_\infty)}{\Gamma_0} = \varepsilon_\infty + (1 - \varepsilon_\infty)\left(\frac{|\Omega_0 - k_0|^2 + \Delta^2}{2\Delta|\Omega_0 - k_0|} - 1\right), \qquad (12)$$

on the dotted line, which is given by

$$\eta^{(m)} = [|\Omega_0 - k_0| - \Delta]/\pi. \qquad (13)$$

We find that $\eta^{(b)}$ and $\eta^{(m)}$ do not depend on ε_∞.

We finally discuss the significance of our results, for experiments on the QZE or the AZE, and for *general* experiments. As mentioned earlier, indirect measurements are necessary, which have not been performed yet, for the complete experimental verification of the QZE and AZE. For such experiments, formula (8) gives the necessary condition: Γ of Eq. (8) should significantly differ from the free decay rate Γ_0. In the Lorentzian case, this can be decomposed into the following conditions: (i) τ_r should be short enough; $\tau_r \lesssim \tau_j$ (which is a well-known condition), (ii) η should not be close to the phase boundary (11), and (iii) ε_∞ should be so small that the first term of Eq. (10) becomes dominant. Moreover, the QZE or the AZE should be chosen according to the phase diagram, Fig. 3; e.g., the AZE is most detectable on the dotted line. On the other hand, in general experiments, one usually wants to *avoid* the QZE and the AZE in order to get correct results. Considering recent rapid progress of experimental techniques and diversification of experimental objects, we expect that the QZE or the AZE would slip in results of advanced experiments not designed to detect it. To avoid the QZE and the AZE, one must design the experimental setup to break at least one of the above conditions. For example, when performing an experiment with a high time resolution such that $\tau_r \lesssim \tau_j$, then Eqs. (9) and (10) suggest that ε_∞ should be *increased* [17]. If ε_∞ cannot be increased to keep the sensitivity of such high-speed measurement, then one should adjust parameters in such a way that Eq. (11) is satisfied, or, one should calibrate the observed value using our results, such as Eqs. (9) and (10), to obtain the free decay rate.

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